

**Subject code-PCI7D002**

**ADVANCED DESIGN OF REINFORCED**  
**CONCRETE STRUCTURES**

**Module-I**

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## **SYLLABUS**

### **Module-I - Earthquake Engineering**

- Introduction to Earthquake Engineering
- Cyclic behaviour of concrete and reinforcement
- Computation of earthquake forces on building frame using Seismic Coefficient Method as per IS 1893-2002
- Base shear and storey shear calculation for multi-storeyed building frames
- Significance of ductility
- Ductility of beam
- Design and detailing for ductility
- Simple problems based on above concept as per IS 13920

### **Module-II - Retaining walls**

- Retaining walls
- Forces acting on retaining wall
- Stability requirement
- Design of Cantilever and Counterfort Retaining walls

### **Module-III - Bridges**

- Introduction to bridges:
  - Classification and components of a standard bridge
  - Economical span
  - Location of piers and abutments
  - Vertical clearance above HFL
  - Scour depth
  - Choice of bridge type
- Standard Loadings for Road Bridges, Impact effect and impact factor calculation for RCC and steel bridges
- Design of single vent rectangular slab culvert

### **Module-IV- Foundations**

- Design of Foundations:
  - Design of Rectangular and Trapezoidal Combined footing

### **References for Module-I**

1. A. K. Jain, “Reinforced Concrete: Limit State Design”, Nem Chand Brothers, Roorkee, 7<sup>th</sup> edition, 2016.
2. B.C. Punmia, A. K. Jain and A. K. Jain, “Limit state design of reinforced concrete (As per IS 456 : 2000)”, Laxmi Publications, 2016.
3. C.V.R. Murthy, Rupen Goswami, A.R. Vijayanarayanan, Vipul V Mehta, Some Concepts in Earthquake Behaviour of Buildings, Gujarat State Disaster Management Authority, Govt. Of Gujarat, 2014.
4. S. K. Duggal, “Earthquake-Resistant Design of Structures”, Oxford University Press, 2<sup>nd</sup> edition, 2017.

**Detailed Course of Module-I**

<b>Sl. No.</b>	<b>Module</b>	<b>Lecture No.</b>	<b>Content</b>
<b>1</b>	<b>I</b>	<b>1</b>	<b>Introduction to Earthquake Engineering</b>
<b>2</b>		<b>2</b>	<b>Earthquake-resistant Buildings</b>
<b>3</b>		<b>3</b>	<b>Cyclic behaviour of concrete and reinforcement</b>
<b>4</b>		<b>4</b>	<b>Computation of earthquake forces on building frame using Seismic Coefficient Method as per IS 1893-2002 (Base shear and storey shear calculation for multi-storeyed building frames)</b>
<b>5</b>		<b>5</b>	<b>Significance of ductility Ductility of beam Design and detailing for ductility Simple problems based on above concept as per IS 13920</b>



# **LECTURE – 1**

**INTRODUCTION TO EARTHQUAKE ENGINEERING**

# 1. INTRODUCTION TO EARTHQUAKE ENGINEERING

## 1.1. EARTHQUAKE ENGINEERING



Fig.1.1. Building knocked off its foundation by the January 1995 earthquake in Kobe, Japan

- **Earthquake engineering** is a branch of engineering that deals in designing and analyzing structures subjected to seismic loading.
- The effects of earthquakes on people and their environment and methods of reducing those effects are included in the earthquake engineering.
- **Aim of earthquake engineering** is to properly design and construct the structures in accordance with building codes, so as to minimize damage due to earthquakes.

## 1.2. EARTHQUAKE

- **Earthquake** is defined as, ‘Ground shaking and radiated seismic energy caused mostly by sudden slip on a fault, volcanic or any sudden stress change in the earth’.
- **An earthquake** is the sudden shaking of the ground caused by the passage of seismic waves through Earth's rocks.



Fig. 1.2. Effects of earthquake

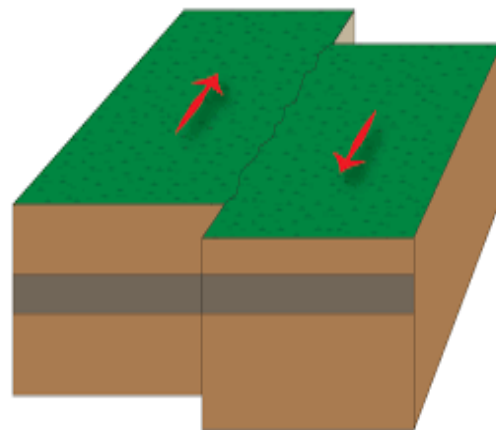


Fig. 1.3. Strike-slip fault

### 1.3. TERMS RELATED TO EARTHQUAKE

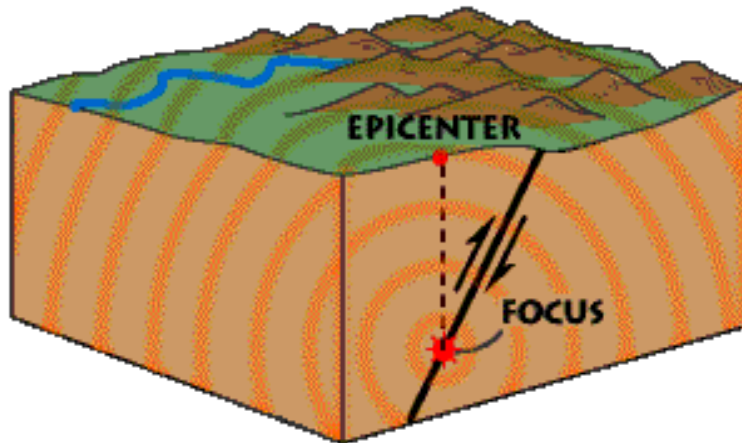


Fig. 1.4. Focus and epicentre of an earthquake

#### Seismology:

- **Seismology** is the study of the generation, propagation and measurement of seismic waves through earth and the sources that generate them.
- **Seismology** is the branch of geophysics which deals with the occurrence of earthquakes and related phenomena on the planet earth.

#### Hypocenter or Focus:

- An earthquake is generally due to some disturbance or displacement in the rocks at some depth below the surface of the Earth.
- Shock waves originate from that place or point of disturbance and then travel in all directions causing the vibrations.
- The place or point of origin of an earthquake in the interior of the earth is known as **focus or hypocenter (Fig. 1.4)**.
- In modern seismology, focus signifies a zone rather than a point of origin. It may lie from a few hundred meters to hundreds of kilometres below the surface.

#### Epicenter:

- The point or place on the surface vertically above the focus of a particular earthquake is termed as its **epicentre (Fig. 1.4)**.
- It is that (geographical) place on the surface of the earth where the vibration from a particular earthquake reaches first of all.
- It is often the location of maximum damage in that event.

**Magnitude:**

- **Magnitude** is the term expressing the rating of an earthquake on the basis of amplitude of seismic waves recorded as seismograms.
- The method (of determining rating of an earthquake) was first used by Charles F Richter in 1935 who developed a scale of magnitude for local use on the basis of study of records of earthquakes of California, USA.
- Subsequently that scale was improved upon and is presently used internationally for describing the size of an earthquake.
- In precise terms and as understood today, the **Richter Magnitude** is the logarithm to the base of 10 of the maximum seismic wave amplitude recorded on a seismograph at a distance of 100 km from the epicentre of a particular earthquake.

**Intensity:**

- **Intensity** is the rating of the effects of an earthquake at a particular place based on the observations of the affected areas, using a descriptive scale like Modified Mercalli Scale.

# **LECTURE – 2**

**EARTHQUAKE-RESISTANT BUILDINGS**

## 2. EARTHQUAKE-RESISTANT BUILDINGS

### 2.1 DYNAMIC ACTIONS ON BUILDINGS – WIND versus EARTHQUAKE

- **Dynamic actions** are caused on buildings by **both wind** and **earthquakes**. But, design for wind forces and for earthquake effects are distinctly different.
- The intuitive philosophy of structural design uses force as the basis, which is consistent in **wind design**, wherein the building is subjected to a pressure on its exposed surface area; this is *force-type loading*.
- However, in **earthquake design**, the building is subjected to random motion of the ground at its base (**Fig. 2.1**), which induces inertia forces in the building that in turn cause stresses; this is *displacement-type loading*.
- Another way of expressing this difference is through the **load-deformation curve** of the building – the demand on the building is *force* (i.e., *vertical axis*) in force-type loading imposed by wind pressure, and *displacement* (i.e., *horizontal axis*) in displacement-type loading imposed by earthquake shaking.

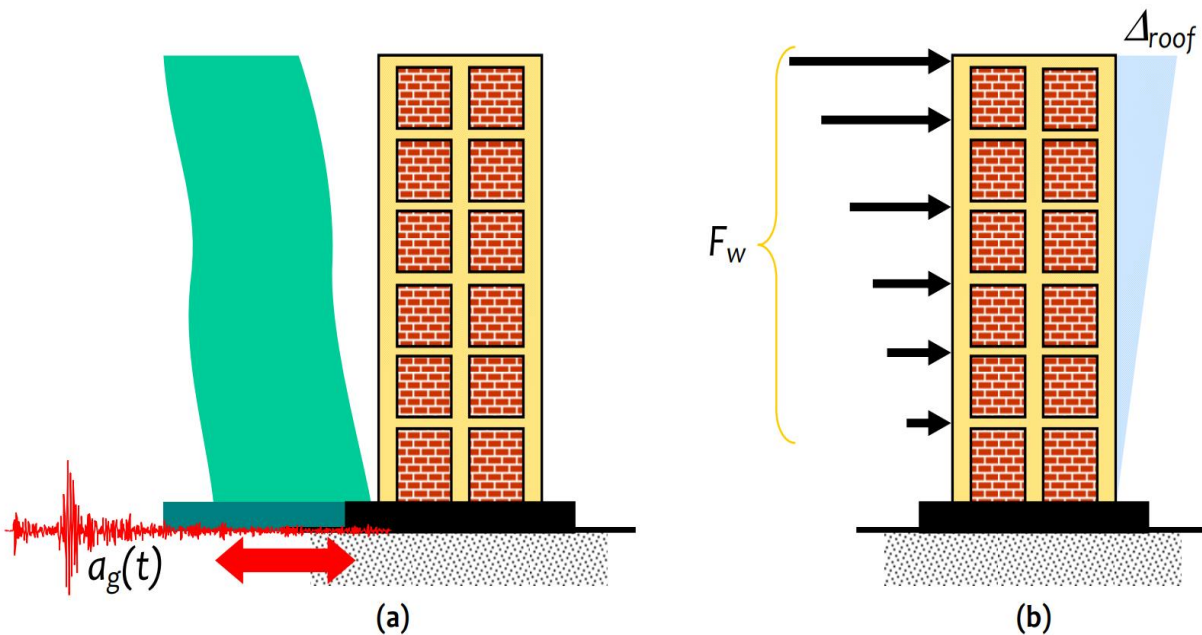


Fig. 2.1. Difference in the design effects on a building during natural actions of (a) Earthquake *Ground Movement* at base, and (b) Wind *Pressure* on exposed area

- **Wind force** on the building has a non-zero mean component superposed with a relatively small oscillating component (**Fig. 2.2**).
- Thus, under wind forces, the building may experience small fluctuations in the stress field, but reversal of stresses occurs only when the direction of wind reverses, which happens only over a large duration of time.

- On the other hand, the motion of the ground during the **earthquake** is cyclic about the neutral position of the structure.
- Thus, the stresses in the building due to seismic actions undergo many complete reversals and that too over the small duration of earthquake.

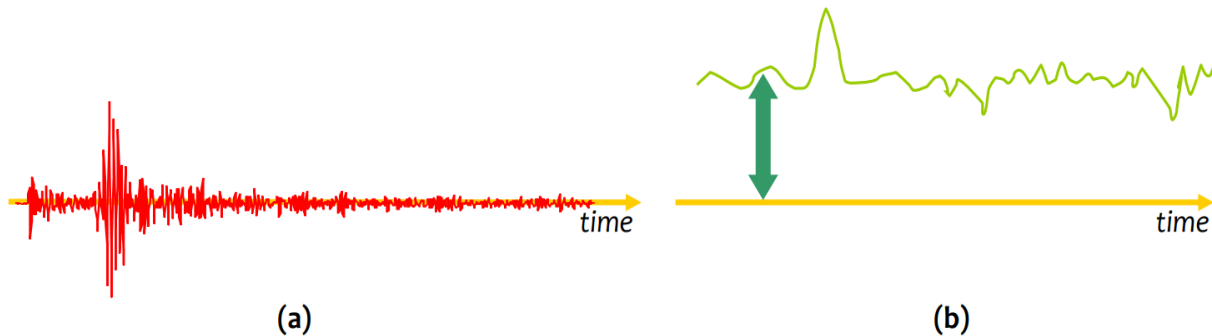


Fig. 2.2. Nature of temporal variations of design actions: (a) Earthquake Ground Motion – zero mean, cyclic, and (b) Wind Pressure – non-zero mean, oscillatory

## 2.2. BASIC ASPECTS OF SEISMIC DESIGN

- The mass of the building being designed controls seismic design in addition to the building stiffness, because earthquake induces inertia forces that are proportional to the building mass.
- Designing buildings to behave elastically during earthquakes without damage may render the project economically unviable.
- As a consequence, it may be necessary for the structure to undergo damage and thereby dissipate the energy input to it during the earthquake.
- Therefore, the **performance criteria (earthquake resistant design philosophy)** implicit in most earthquake codal provisions (Jain 1980, 1996, IS:1893, IBC 2006) require that structure should be able to resist (**Fig. 2.3**):

(a) **earthquakes of minor (and frequent) intensity** with no damage to structural and non-structural elements. A structure would be expected to resist such frequent but minor shocks within its elastic range of stresses;

(b) **earthquakes of moderate intensity** with minor damage to structural elements, and some damage to non-structural elements. With proper design and construction, it is believed that structural damage due to the majority of earthquakes will be limited to repairable damage; and

(c) **earthquakes of severe (and infrequent) intensity** with damage to structural elements, but with no collapse (to save life and property inside/adjoining the building). Severe structural damage is expected.

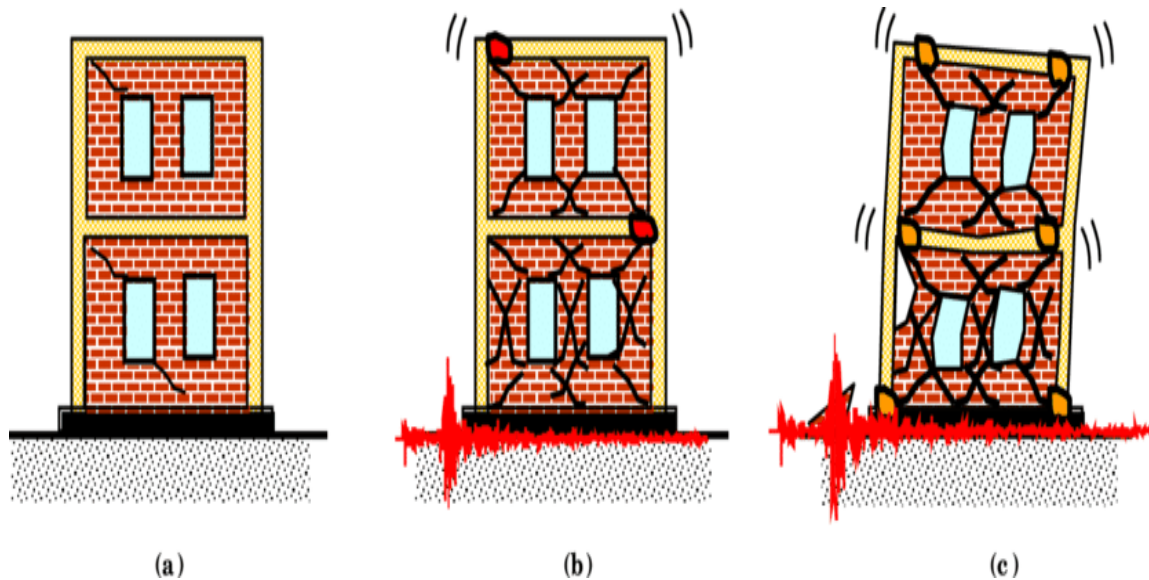


Fig. 2.3. Earthquake-Resistant Design Philosophy for buildings: (a) Minor (Frequent) Shaking – No/Hardly any damage, (b) Moderate Shaking – Minor structural damage, and some non-structural damage, and (c) Severe (Infrequent) Shaking – Structural damage, but no collapse

Table 2.1. Seismic design criteria

Earthquake	Desired behaviour	Controlling parameter
<b>Minor</b>	No damage to structural components	Control deflection by providing <i>stiffness</i>
<b>Moderate</b>	No significant structural damage, minor cracks in beams and columns, Response should be predominantly elastic	Avoid yielding of members or permanent damage by providing <i>strength</i>
<b>Severe, Catastrophic</b>	No collapse of the system which could cause loss of life	Allow structure to enter into inelastic range and absorb energy by providing <i>ductility</i>

- Therefore, buildings are designed only for a fraction (~8-14%) of the force that they would experience, if they were designed to remain elastic during the expected strong ground shaking, and thereby permitting damage.
- But, sufficient initial stiffness is required to be ensured to avoid structural damage under minor shaking.
- Thus, seismic design balances reduced cost and acceptable damage, to make the project viable.
- This careful balance is arrived based on extensive research and detailed post-earthquake damage assessment studies.
- A wealth of this information is translated into precise seismic design provisions. In contrast, structural damage is not acceptable under design wind forces.



- For this reason, design against earthquake effects is called as **earthquake-resistant design** and **not earthquake-proof design**.

## 2.3. THE FOUR VIRTUES OF EARTHQUAKE RESISTANT BUILDINGS

For a building to perform satisfactorily during earthquakes, it must meet the philosophy of earthquake-resistant design discussed in **Section 2.2**.

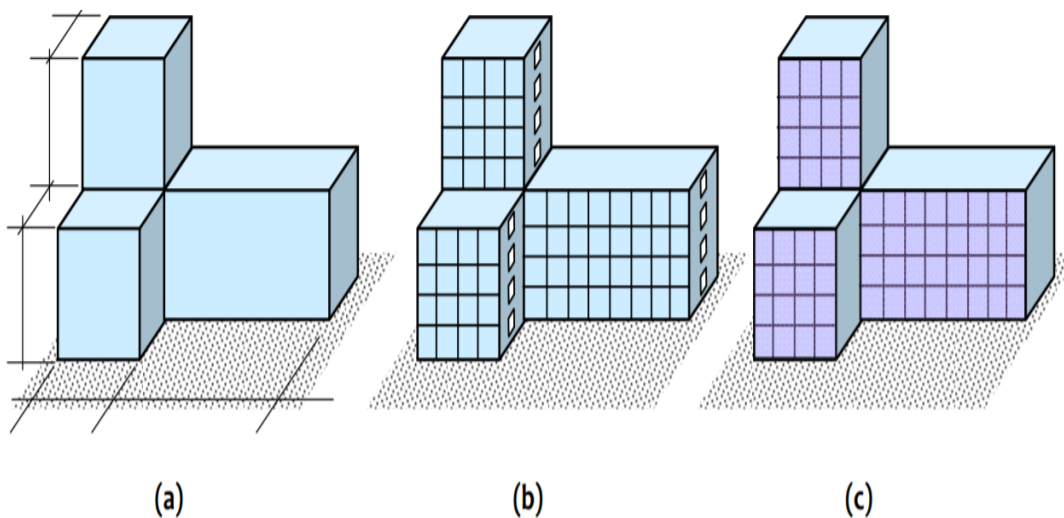
### 2.3.1 Characteristics of Buildings

- There are **four aspects of buildings** that architects and design engineers work with to create the earthquake-resistant design of a building, namely **seismic structural configuration, lateral stiffness, lateral strength and ductility**, in addition to other aspects like form, aesthetics, functionality and comfort of building.
- **Lateral stiffness, lateral strength and ductility of buildings** can be ensured by strictly following most seismic design codes.
- But, **good seismic structural configuration** can be ensured by following coherent architectural features that result in good structural behaviour.

#### (i) Seismic Structural Configuration

Seismic structural configuration entails three main aspects, namely

- (a) geometry, shape and size of the building,
- (b) location and size of structural elements, and
- (c) location and size of significant non-structural elements (**Fig. 2.4**).



**Fig. 2.4.** Components of seismic structural configuration: (a) overall geometry, (b) structural elements (e.g., moment resisting frames and structural walls), and (c) significant non-structural elements (e.g., façade glass)

- Buildings can be placed in two categories, namely **simple and complex (Fig. 2.5)**.
- Buildings with rectangular plans and straight elevation stand the best chance of doing well during an earthquake, because inertia forces are transferred without having to bend due to the geometry of the building (**Fig. 2.5a**).
- But, buildings with setbacks and central openings offer geometric constraint to the flow of inertia forces; these inertia force paths have to bend before reaching the ground (**Fig. 2.5b, 2.5c**).

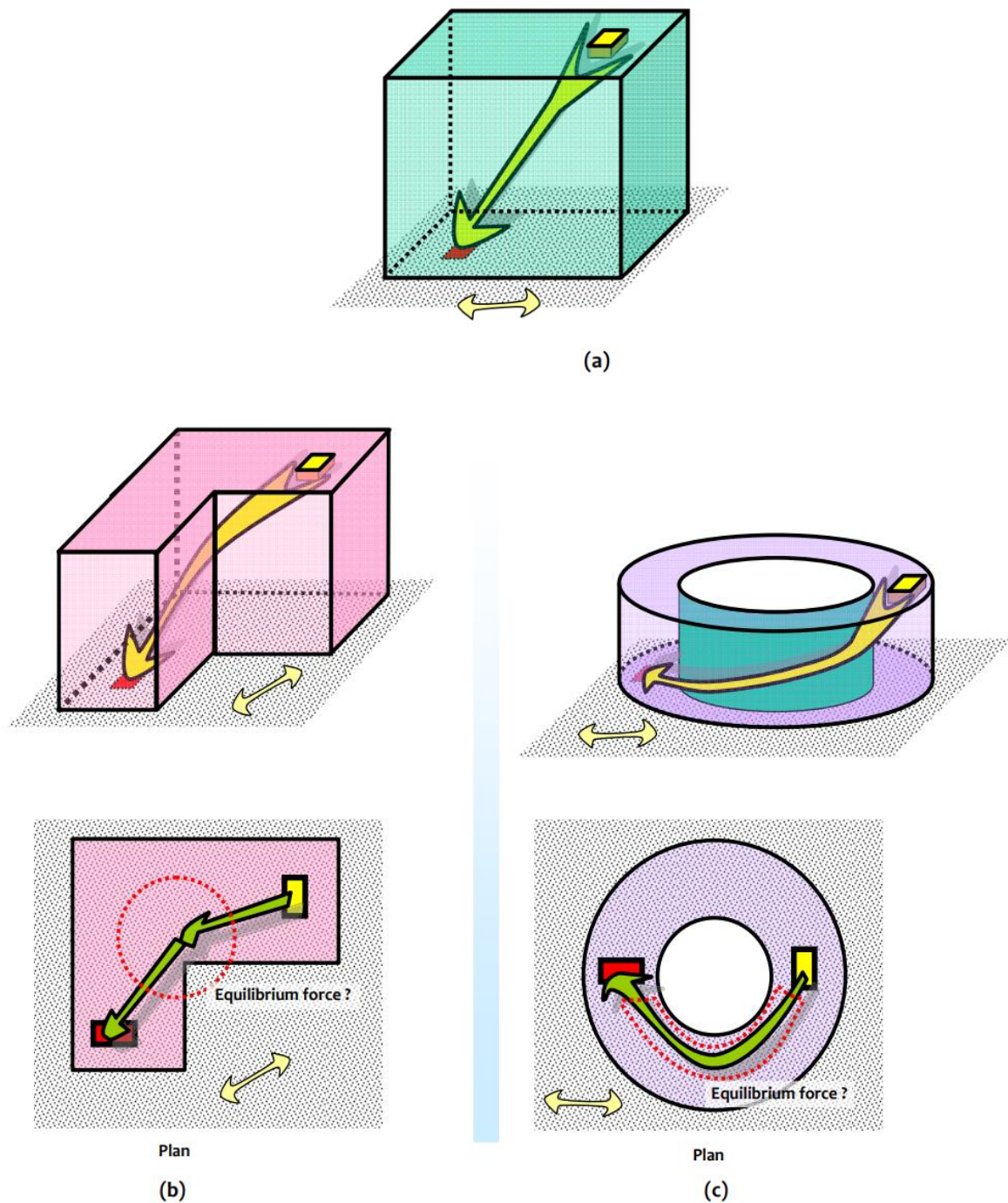


Fig. 2.5. Classification of buildings: (a) Simple, and (b), (c) Complex

## (ii) Structural Stiffness, Strength and Ductility

- The next three overall properties of a building, namely **lateral stiffness**, **lateral strength** and **ductility**, are illustrated in **Fig. 2.6**, through the lateral load - lateral deformation curve of the building.
- **Lateral stiffness** refers to the initial stiffness of the building, even though stiffness of the building reduces with increasing damage.
- **Lateral strength** refers to the maximum resistance that the building offers during its entire history of resistance to relative deformation.
- **Ductility** towards lateral deformation refers the ratio of the maximum deformation and the idealised yield deformation.
- The maximum deformation corresponds to the maximum deformation sustained by it, if the load-deformation curve does not drop, and to 85% of the ultimate load on the dropping side of the load-deformation response curve after the peak strength or the lateral strength is reached, if the load-deformation curve does drop after reaching peak strength.

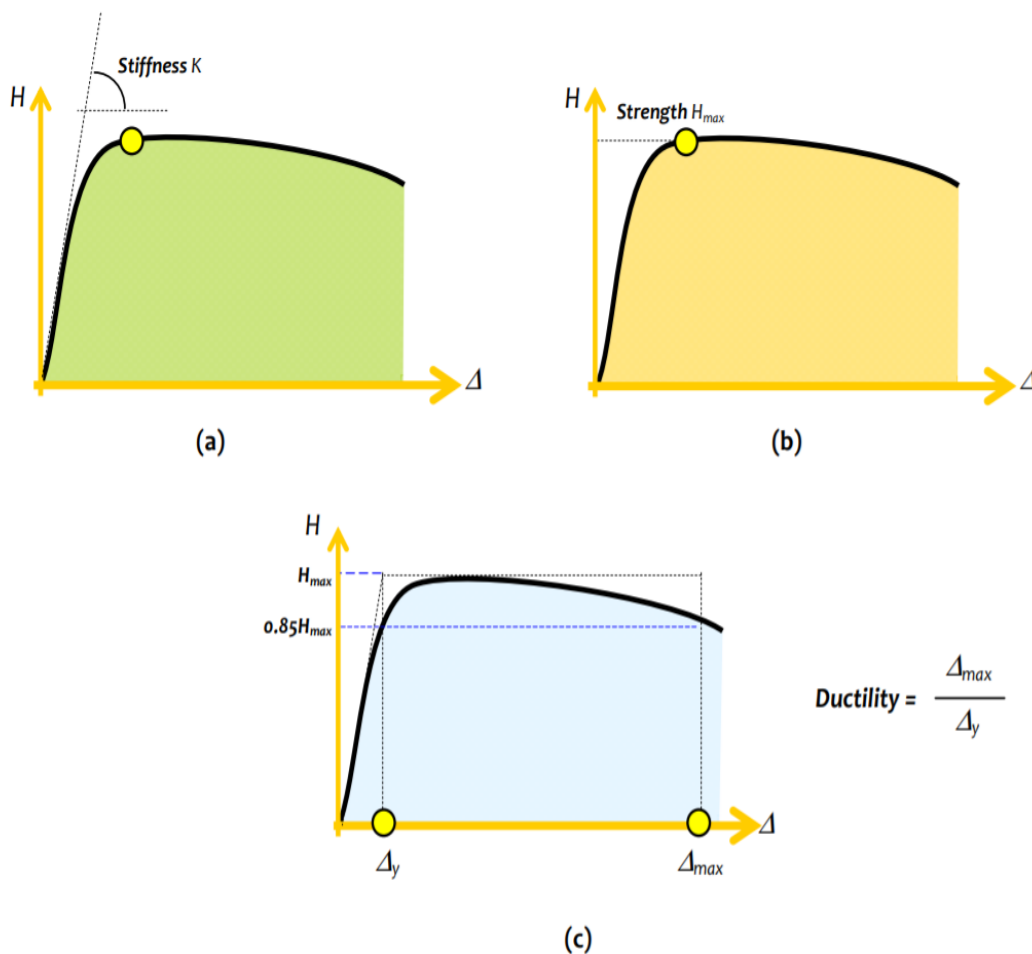


Fig. 2.6: Structural Characteristics: Overall load deformation curves of a building, indicating (a) lateral stiffness, (b) lateral strength, and (c) ductility towards lateral

### 2.3.2 What are the Four Virtues?

- All buildings are vertical cantilevers projecting out from the earth's surface.
- Hence, when the earth shakes, these cantilevers experience whiplash effects, especially when the shaking is violent. Hence, special care is required to protect them from this jerky movement.
- Buildings intended to be earthquake-resistant have competing demands.
- **Firstly**, buildings become expensive, if designed not to sustain any damage during strong earthquake shaking.
- **Secondly**, they should be strong enough to not sustain any damage during weak earthquake shaking.
- **Thirdly**, they should be stiff enough to not swing too much, even during weak earthquakes.
- And, **fourthly**, they should not collapse during the expected strong earthquake shaking to be sustained by them even with significant structural damage.
- These competing demands are accommodated in buildings intended to be earthquake resistant by incorporating **four desirable characteristics** in them.
- **These characteristics, called the four virtues of earthquake-resistant buildings, are:**
  - i. **Good seismic configuration**, with no choices of architectural form of the building that is detrimental to good earthquake performance and that does not introduce newer complexities in the building behaviour than what the earthquake is already imposing;
  - ii. **At least a minimum lateral stiffness** in each of its plan directions (uniformly distributed in both plan directions of the building), so that there is no discomfort to occupants of the building and no damage to contents of the building;
  - iii. **At least a minimum lateral strength** in each of its plan directions (uniformly distributed in both plan directions of the building), to resist low intensity ground shaking with no damage, and not too strong to keep the cost of construction in check, along with a minimum vertical strength to be able to continue to support the gravity load and thereby prevent collapse under strong earthquake shaking; and
  - iv. **Good overall ductility** in it to accommodate the imposed lateral deformation between the base and the roof of the building, along with the desired mechanism of behaviour at ultimate stage.
- Behaviour of buildings during earthquakes depend critically on these four virtues. Even if any one of these is not ensured, the performance of the building is expected to be poor.

### 2.3.3 Who controls the Four Virtues?

- Both the **architect and the engineer** work together to create the best design with good interaction at all stages of the process of the design of the building.
- Here, the **architect** brings in perspectives related to form, functionality, aesthetics and contents, while the **engineer** brings the perspectives of safety and desired earthquake performance during an expected earthquake.

### 2.3.4 How to achieve the Four Virtues?

- The **four virtues** are achieved by **inputs** provided at all stages of the development of the building, namely **planning, design, construction and maintenance**.
  - Each building to be built is only one of the kind ever, and no research and testing is performed on that building, unlike factory made products like aircrafts, ships and cars.
  - The owner of the building trusts the professionals (i.e., architect and engineer) to have done due diligence to design and construct the building.
  - Thus, **professional experience** is essential to be able to conduct a safe design of the building, because it affects the safety of persons and property.
- **Traditionally**, in countries that have **advanced earthquake safety initiatives**, governments have played critical role through the enforcement of techno-legal regime, wherein the municipal authorities arrange to examine, if all requisite **technical inputs** have been met with to ensure **safety** in the building, before allowing the building to be built, the construction to be continued at different stages for the users to occupy the building.
- These **stages** are: **(1) conceptual design stage, (2) design development stage through peer review of the structural design, (3) construction stage through quality control and quality assurance procedures** put in place.
- Senior professionals (both architects and engineers) are required to head the team of professionals to design a building; these senior professionals should have past experience of having designed buildings to resist strong earthquakes under the tutelage of erstwhile senior professionals.

## 2.4. STRUCTURAL SYSTEMS

- A building is subjected to gravity loads such as **dead loads and live loads**, and **lateral loads such as wind or earthquake loads**.
- These loads are required to be transferred safely to the soil below through a system of interconnected structural members.

- Connections between beam and column or beam, column and wall or bracing member may be simple or moment resistant based on economy or other practical considerations.
- In tall buildings, the biggest challenge comes from satisfying the serviceability limit state, constructability, durability as well as economy.
- A structural system needs to be evolved to satisfy all the preceding requirements of the structure.

#### ❖ **Gravity load resisting system**

**Purpose-** To transfer gravity loads applied at floor levels down to the foundation level

##### ➤ **Direct Path Systems**

- Slab supported on load bearing walls
- Slab supported on columns

##### ➤ **Indirect Multi Path Systems**

- Slab supported on beams
- Beams supported on other beams
- Beams supported on walls or columns

#### ❖ **Lateral load resisting system**

**Purpose-** To transfer lateral loads applied at any location in the structure down to the foundation level.

##### ➤ **Single System**

- Moment resisting frames
- Braced frames
- Shear walls
- Tubular systems

##### ➤ **Dual System**

- Shear wall-frames
- Tube + frame + shear wall

- **A structural system** may be classified as follows:

- i. Load bearing wall system
- ii. Moment resisting frame system
- iii. Flexural (shear) wall
- iv. Dual frame system
- v. Tube system

### i. Load bearing wall system

- It is a system which is designed for gravity as well as for lateral loads.
- Under lateral loads the walls act like cantilevers.
- The walls and partition wall supply in-plane lateral stiffness and stability to resist wind and earthquake loading.
- This system lacks in providing redundancy for the vertical and lateral load supports, i.e. if a wall fails, the vertical loads as well as lateral loads carrying capacity is eliminated leading to instability.
- Clause 8.2.1 of IS: 4326-1993 restricts the use of such structural systems to 3 storeys in seismic zone V and 4 storeys in other zones.

### ii. Moment resisting frame system

- It is a system in which beams, columns and joints are capable of resisting vertical and lateral loads, primarily by flexure.
- The beam-column joint is the most crucial component. The moment distribution among beams and columns takes place through joints. If a beam-column joint fails, the whole structural system will fail.
- In a moment resisting frame, relative stiffness of beams and columns is very important.
- A frame may be designed as having weak column-strong beam proportions or strong column-weak column proportions.
- Under lateral loads, the failure mechanism in two types of frames are shown in the **Fig. 2.7**.
- A plastic hinge will form at the ends of a column or a beam depending upon which is weaker.
- The reasons for having **strong column-weak beam** arrangement are:
  - Failure of a column means the collapse of the entire building.
  - In a weak-column structure, plastic deformation is concentrated in a particular storey, as shown in **Fig. 2.7(b)**, and a relatively large ductility factor is required.
  - In both shear and flexural failure of columns, degradations are greater than those in the yielding of beams.

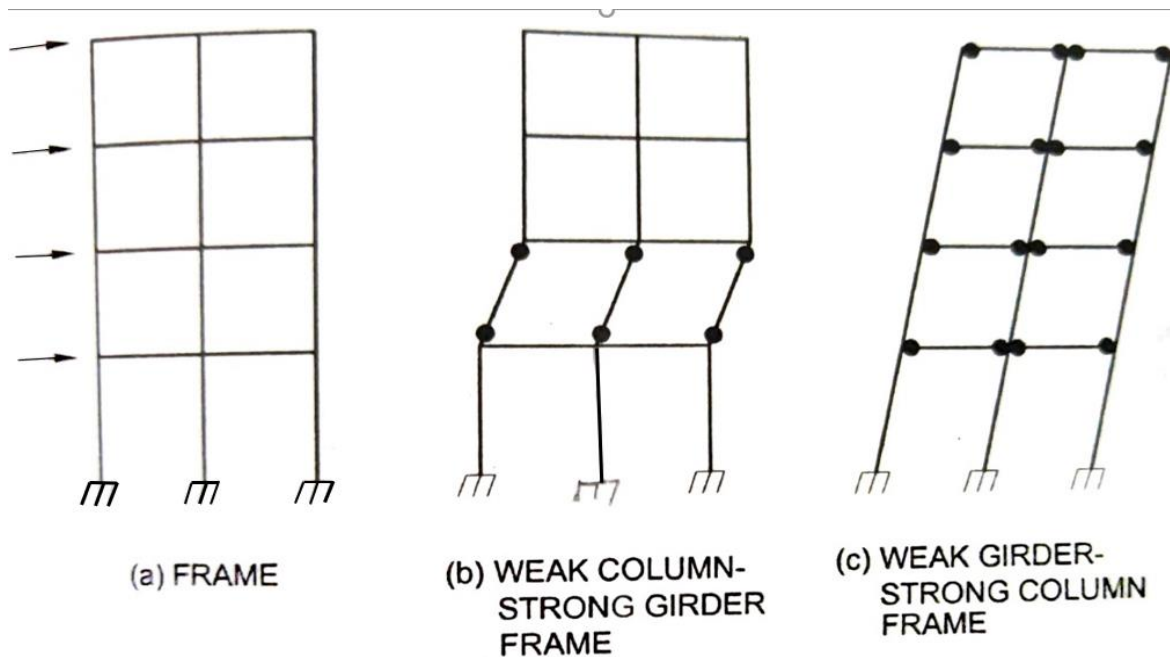


Fig. 2.7. Failure mechanisms in rigid frame

- **Types of moment resisting space frames**

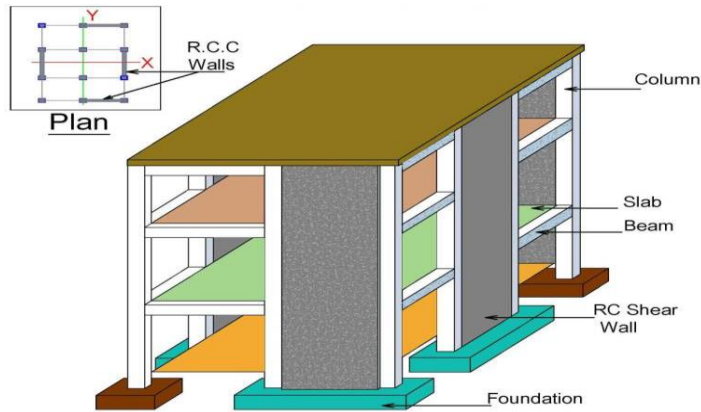
- **Ordinary moment resisting frame** - It is a space frame capable of carrying all vertical and horizontal loads, by developing bending moments in the members and at joints, but not meeting the special detailing requirements for ductile behavior.
- **Ductile/ Special moment resisting frame** - It is a moment resisting frame detailed to provide ductile behaviour and comply with the requirements given in **IS 4326 or IS 13920 or SP 6**. A frame of continuous construction comprising flexural members and columns designed and detailed to accommodate reversible lateral displacements after the formation of plastic hinges (without the decrease in strength) is known as **ductile moment resisting frame**.

### iii. Flexural (shear) wall

- It is a reinforced concrete wall designed to resist lateral forces in its own plane.
- Shear walls (**Fig. 2.8.**) are reinforced concrete walls cantilevering vertically from the base (i.e. foundations) designed and detailed to be ductile so as to resist seismic forces and to dissipate energy through flexural yielding at one or more plastic hinges.
- Shear walls should extend from foundations either to the top of the building or to a lesser height as required from the design consideration.
- Studies show that shear walls of height about 85 percent of total height of building are advantageous.



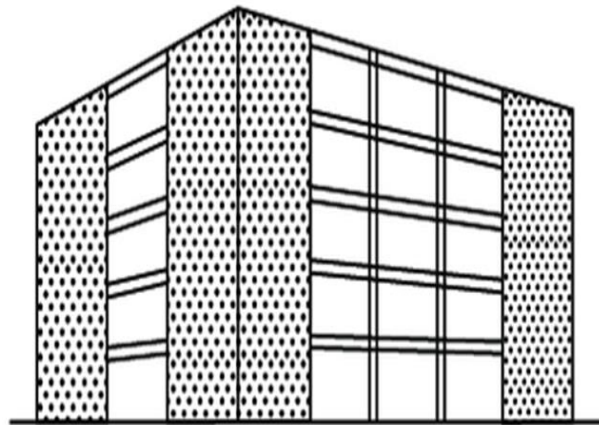
- A shear wall building is normally quite rigid as compared to a frame structure.



**Fig. 2.8. Reinforced Concrete Shear Wall**

#### **iv. Dual frame system (Fig. 2.9.)**

- It is a system which consists of moment resisting frames either braced or with shear walls.



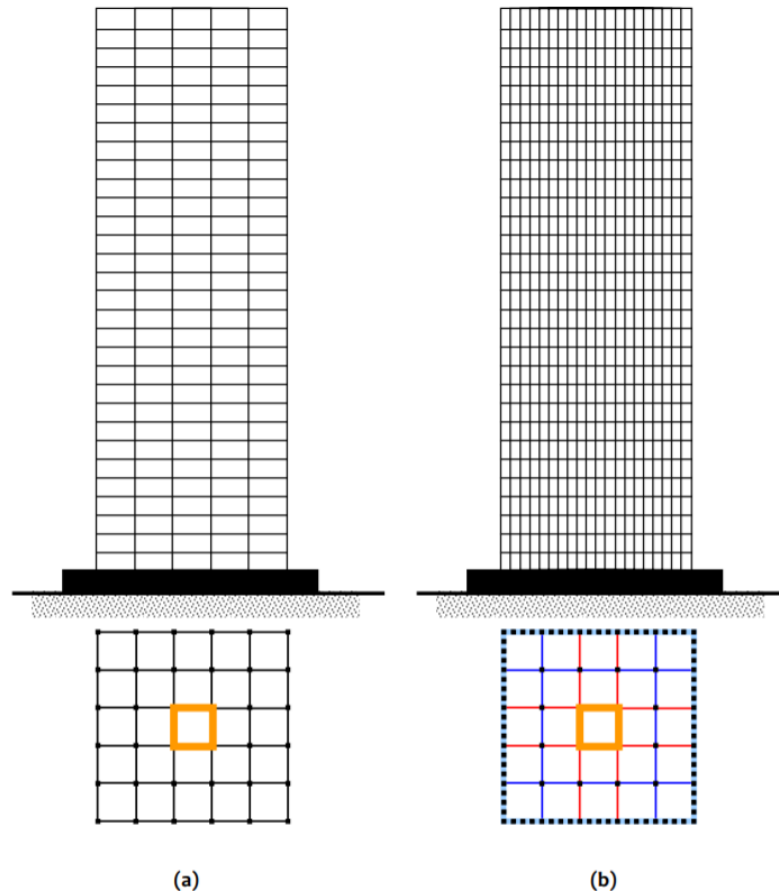
**Fig. 2.9. Frames with Shear Wall**

- This system has following features:
  - A moment resisting frame provide support for gravity load.
  - The moment resisting frames are designed to resist at least 25 percent of the base shear including torsion effects.
  - Flexural walls, that is shear walls or braced frames must resist the total required lateral force in accordance with lateral stiffness considering the interaction of the walls and the frames as a single system.

#### **v. Tube system (Fig. 2.10(b))**

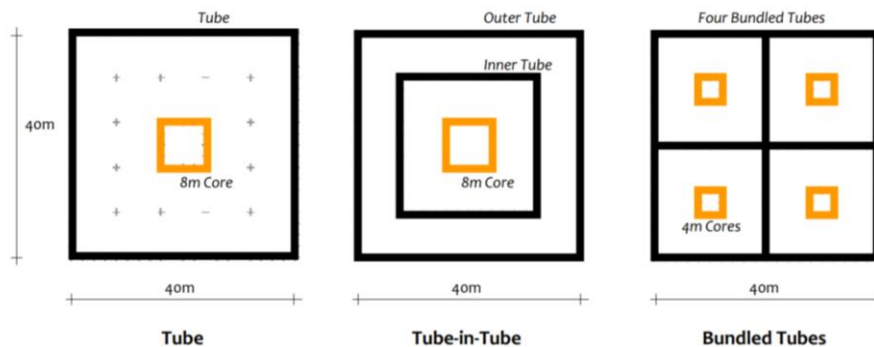
- It is a fully three-dimensional system that utilizes the entire building perimeter to resist lateral loads.

- It is a structural system consisting of closely spaced exterior columns tied at each floor level with relatively deep spandrel beams. Thus, it creates the effect of a hollow concrete tube perforated by openings for the windows.



**Fig. 2-10. (a) Traditional Frame System with a central core, and (b) Tube System**

- The exterior columns are generally spaced between 1.25 m to 3 m.
- The spandrel beams interconnecting the closely spaced columns have a depth varying from 60 cm to 1.25 m and width varying from 25 cm to 1 m.
- This system is very effective in controlling the lateral displacements in very tall buildings.
- Three basic types of tube structures are shown in **Fig. 2.11**.



**Fig. 2.11. Three basic types of tube systems**

# **LECTURE – 3**

**CYCLIC BEHAVIOUR OF CONCRETE AND REINFORCEMENT**

### 3. CYCLIC BEHAVIOUR OF CONCRETE AND REINFORCEMENT

#### 3.1. BRITTLE AND DUCTILE BEHAVIOUR OF MATERIALS

- The term **ductility** implies the ability of a material to sustain significant deformation prior to collapse.
- A **brittle** material is the one that fails suddenly upon attaining the maximum load.
- Typical **force-deformation relationships** for brittle and ductile materials are shown in **fig. 3.1.**

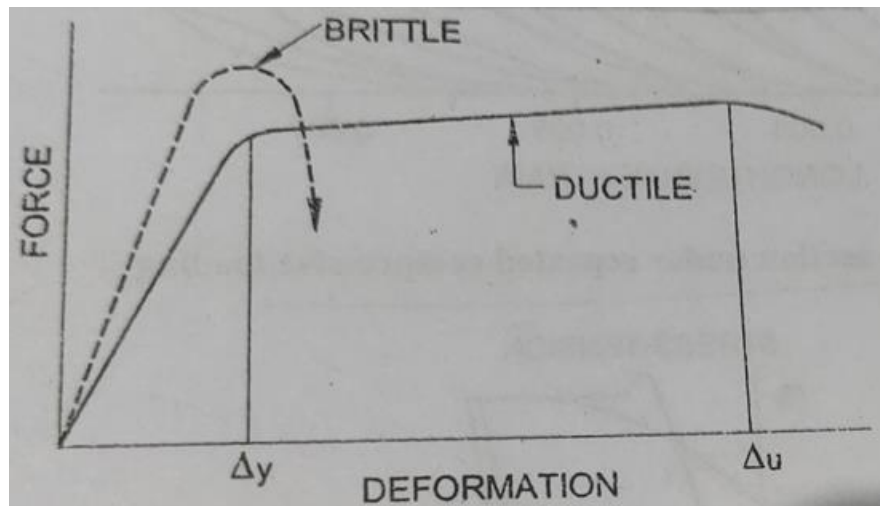


Fig. 3.1. Brittle and ductile force-deformation behaviour

#### 3.2. CYCLIC BEHAVIOUR OF PLAIN CONCRETE

- Plain concrete comes under the category of brittle materials.
- During the first cycle, the stress strain curve is the same as that obtained from static tests. If the specimen is unloaded and reloaded in compression, stress strain curves similar to those shown in **fig. 3.2.** are obtained.
- It can be seen that slope of the stress strain curves as well as the maximum attainable stress decrease with the number of cycles. Thus, **the stress strain relationship for plain concrete subjected to repeated compressive loads is cycle dependent.**
- The decrease in stiffness and strength of plain concrete is due to the formation of cracks.
- The compressive strength of concrete depends on the rate of loading. As the rate of loading increases, the compressive strength of concrete increases but the strain at the maximum stress decreases.
- Plain concrete cannot be subjected to repeated tensile loads since its tensile strength is practically zero.

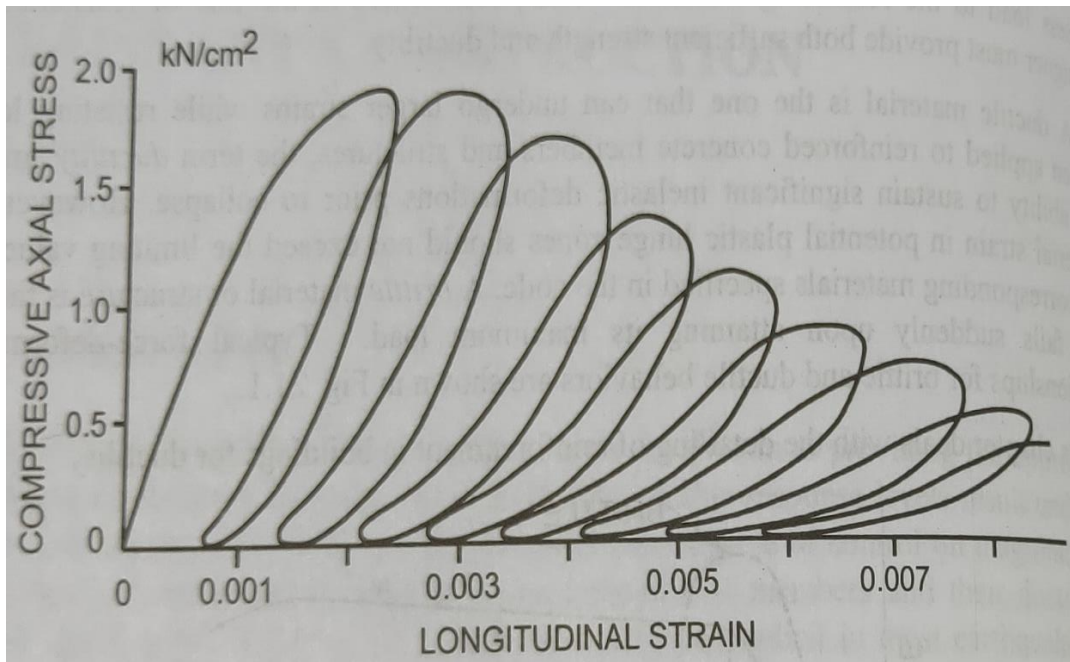


Fig. 3.2. Plain concrete section under repeated compressive loading

### 3.3. CYCLIC BEHAVIOUR OF REINFORCING STEEL

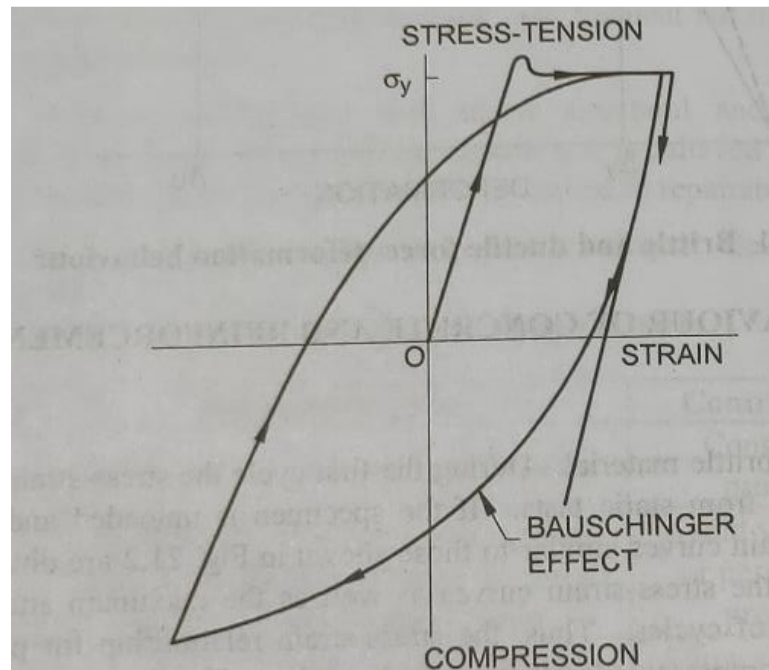


Fig. 3.3. Hysteresis behaviour of reinforcing steel

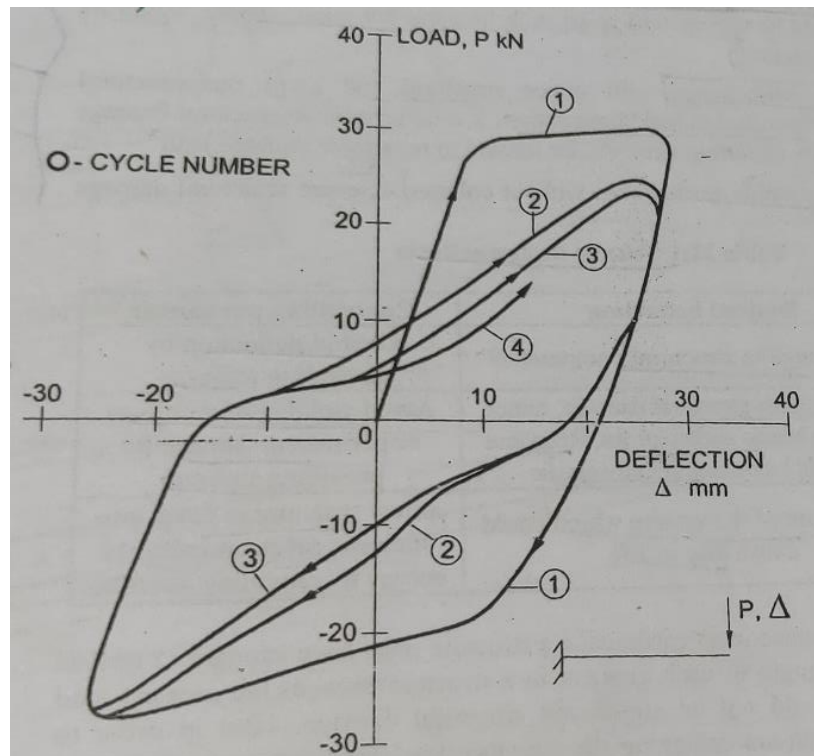
- Reinforcing steel has much more ductility than plain concrete.
- The ultimate strain in mild steel is of the order of 25% whereas, in concrete it is of the order of 0.3%.
- In the first cycle, the reinforcing steel shows stress strain curve similar to that obtained in the static test. After the specimen has reached its yield level and direction of load is reversed,

that is, unloading begins, it can be seen in **fig. 3.3** that the unloading curve is not straight but curvilinear. This curvature in the unloading segment of stress-strain curve is referred to as the **Bauschinger effect** after the discoverer of the phenomenon.

- **Fig. 3.3** shows one complete cycle of loading and unloading which is referred to as a **hysteresis loop**.
- The area within a hysteresis loop exhibits energy absorbed by the specimen in a cycle.
- In subsequent cycles, practically the same path is repeated. Thus, **the stress strain relationship for mild reinforcing steel subjected to repeated reversed loading is cycle independent until the specimen buckles or fails due to fatigue**.
- It is also observed that same hysteresis loops are obtained for a specimen which is first loaded in tension followed by compression as when it is first loaded in compression followed by tension.
- The yield strength of reinforcement is also affected by the rate of loading.

### **3.4. CYCLIC BEHAVIOUR OF REINFORCED CONCRETE**

- Earlier it was noticed that plain concrete can be subjected only to repeated compressive loading cycles and not to repeated tensile loading cycles due to its poor tensile strength. However, reinforcing steel can be subjected to repeated reversible tensile and compressive loading cycles and exhibits stable hysteresis loops. Thus, **the cyclic behaviour of reinforced concrete members is significantly improved due to the presence of reinforcing steel**.
- **Fig. 3.4.** shows typical load deflection curves for a cantilever reinforced concrete beam subjected to reversed cyclic loading. Reinforcing steel is present on both faces since one face is in tension during the first half loading cycle and the other face is in tension during the remaining half of the loading cycle. It can be seen in this figure that slope of a load deflection curve that is stiffness of the beam decreases with number of cycles. Moreover, curves tend to pinch in near zero load. These two effects are distinct characteristics of reinforced concrete beams as well as columns and are referred to as stiffness degradation and pinching effects. The nonlinear behaviour of reinforced concrete is affected mainly by the degree of cracking in concrete, strain hardening and Bauschinger effect in reinforcing steel, effectiveness of bond and anchorage between concrete and reinforcing steel and the presence of high shear. It is not possible in quantity the contribution of each of these parameters towards the nonlinearity of reinforced concrete.



**Fig. 3.4. Hysteresis behaviour of a cantilever beam**

- Since stiffness degradation starts right after the first cycles and progresses rapidly, it becomes still more necessary to improve the capability of reinforced concrete to sustain inelastic deformations in order to avoid its collapse.

# **LECTURE – 4**

**COMPUTATION OF EARTHQUAKE FORCES ON BUILDING FRAME**

**USING SEISMIC COEFFICIENT METHOD AS PER IS 1893-2002**

**(Base shear and storey shear calculation for multi-storeyed building frames)**



#### 4. COMPUTATION OF EARTHQUAKE FORCES ON BUILDING FRAME USING SEISMIC COEFFICIENT METHOD AS PER IS 1893-2002

This lecture also includes **base shear and storey shear calculation for multi-storeyed building frames.**

##### 4.1. IS Codes

Various IS codes required for earthquake design are:

- IS 1893-2002
- IS 4326-1993
- IS 13920-1993

##### 4.2. SEISMIC ZONES

There are four seismic zones i.e. Zone-II, Zone-III, zone-IV and Zone-V.

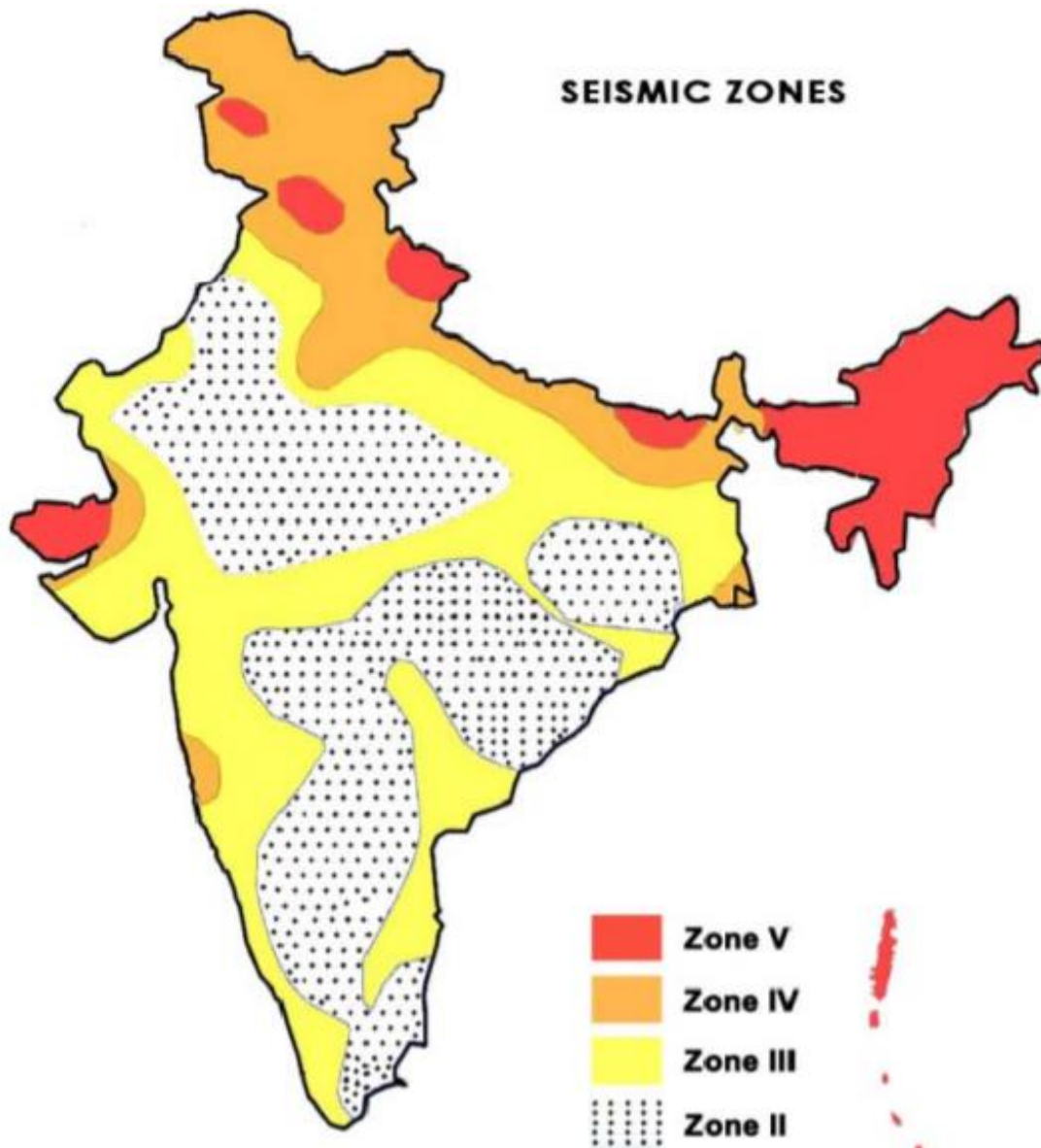


Fig. 4.1. Seismic Zoning map of India - 2007

### **4.3. METHODS TO DETERMINE EARTHQUAKE FORCE IN A BUILDING**

There are two methods to determine earthquake force in a building i.e.

- a) Seismic Coefficient method or Static method or Equivalent lateral force method or Equivalent static method**
- b) Response spectrum method or Modal analysis method or Spectral acceleration method or Dynamic method or Mode superposition method**

**a) Seismic Coefficient method or Static method or Equivalent lateral force method or Equivalent static method**

- Seismic analysis of most structures is still carried out on the assumption that the lateral (horizontal force) is equivalent to the actual (dynamic loading).
- This method requires less effort because except for the fundamental period, the periods and shapes of higher natural modes of vibration are not required.
- The base shear which is the total horizontal force on the structure is calculated on the basis of the structure's mass, its fundamental period of vibration and corresponding shape.
- The base end shear is distributed along the height of the structure in terms of lateral forces, according to the code formula.
- This method is usually conservative for low to medium height buildings with a regular configuration.

**b) Response spectrum method or Modal analysis method or Spectral acceleration method or Dynamic method or Mode superposition method**

- This method is applicable to those structures where modes other than the fundamental one significantly affect the response of structure.

### **4.4. FACTORS IN SEISMIC ANALYSIS**

The factors taken into account in assessing lateral design forces and the design response spectrum are as follows:

- i. Zone factor
- ii. Importance factor
- iii. Response reduction factor
- iv. Fundamental natural period
- v. Design response spectrum

**i. Zone Factor [IS 1893 (Part 1): 2002, Clause 6.4]**

- Seismic zoning assesses the maximum severity of shaking that is anticipated in a particular region.
- The **zone factor (Z)** thus is defined as a factor to obtain the design spectrum depending on the perceived seismic hazard in the zone in which the structure is located.
- The basic zone factors included in the code are reasonable estimate of the effective peak ground acceleration.
- Zone factors as per IS 1893(part 1):2002 are given in **Table 4.1**

**Table 4.1 Zone factor (Z)**

Seismic Zone	II	III	IV	V
Seismic Intensity	Low	Moderate	Severe	Very Severe
Z	0.10	0.16	0.24	0.36

**ii. Importance Factor [IS 1893 (Part 1): 2002, Clause 7.2]**

- The **importance factor (I)** is a factor used to obtain the design seismic force depending upon the functional use of the structure.
- It is customary to recognise that certain categories of building use should be designed for greater levels of safety than the others and this is achieved by specifying higher lateral design forces. Such categories are:
  - a) Buildings which are essential after an earthquake - Hospitals, fire stations, etc.
  - b) Places of assembly - schools, theatres etc
  - c) Structures the collapse of which may endanger lives - nuclear plants, dams, etc.
- The importance factors are given in **Table 4.2**

**Table 4.2 Importance Factors (I)**

Structure	Importance factor (I)
Important services and community buildings such as hospitals, schools, monumental structures, emergency buildings like telephone exchanges, television stations, radio stations, railway stations, fire station buildings, large community halls like cinemas, assembly halls and subway stations, power stations	1.5
All other buildings	1.0

### iii. Response Reduction Factor [IS 1893 (Part 1): 2002, Clause 6.4]

- The basic principle of designing a structure for strong ground motion is that the structure should not collapse but damage to the structural elements is permitted.
- Since a structure is allowed to be damaged in case of severe shaking, the structure should be designed for seismic forces much less than what is expected under strong shaking if the structures were to remain linearly elastic.

**Table 4.3 Response Reduction Factor (R) for building systems**

SI No.	Lateral Load Resisting System	R
(1)	(2)	(3)
<i>Building Frame Systems</i>		
i)	Ordinary RC moment-resisting frame ( OMRF ) <sup>2)</sup>	3.0
ii)	Special RC moment-resisting frame ( SMRF ) <sup>3)</sup>	5.0
iii)	Steel frame with	
	a) Concentric braces	4.0
	b) Eccentric braces	5.0
iv)	Steel moment resisting frame designed as per SP 6 ( 6 )	5.0
<i>Building with Shear Walls<sup>4)</sup></i>		
v)	Load bearing masonry wall buildings <sup>5)</sup>	
	a) Unreinforced	1.5
	b) Reinforced with horizontal RC bands	2.5
	c) Reinforced with horizontal RC bands and vertical bars at corners of rooms and jambs of openings	3.0
vi)	Ordinary reinforced concrete shear walls <sup>6)</sup>	3.0
vii)	Ductile shear walls <sup>7)</sup>	4.0
<i>Buildings with Dual Systems<sup>8)</sup></i>		
viii)	Ordinary shear wall with OMRF	3.0
ix)	Ordinary shear wall with SMRF	4.0
x)	Ductile shear wall with OMRF	4.5
xi)	Ductile shear wall with SMRF	5.0

- **Response reduction factor (R)** is the factor by which the actual base shear force should be reduced, to obtain the design lateral force.
- **Base shear force** is the force that would be generated if the structure were to remain elastic during its response to the design base earthquake (DBE) shaking.
- **Table 4.3** gives response reduction factor for building systems.

**iv. Fundamental Natural Period [IS 1893 (Part 1): 2002, Clause 7.6]**

- The **fundamental natural period** is the first (longest) modal time period of vibration of the structure.
- Because the design loading depends on the building period and the period cannot be calculated until a design has been prepared, IS 1893 (part 1): 2002, Clause 7.6 provides formulae from which  $T_a$  may be calculated.

**7.6.1 The approximate fundamental natural period of vibration (  $T_a$  ), in seconds, of a moment-resisting frame building without brick infill panels may be estimated by the empirical expression:**

$$\begin{aligned} T_a &= 0.075 h^{0.75} && \text{for RC frame building} \\ &= 0.085 h^{0.75} && \text{for steel frame building} \end{aligned}$$

**where**

**$h$  = Height of building, in m. This excludes the basement storeys, where basement walls are connected with the ground floor deck or fitted between the building columns. But, it includes the basement storeys, when they are not so connected.**

**7.6.2 The approximate fundamental natural period of vibration (  $T_a$  ), in seconds, of all other buildings, including moment-resisting frame buildings with brick infill panels, may be estimated by the empirical expression:**

$$T_a = \frac{0.09h}{\sqrt{d}}$$

**where**

**$h$  = Height of building, in m, as defined in 7.6.1;  
and**

**$d$  = Base dimension of the building at the plinth level, in m, along the considered direction of the lateral force.**

**v. Design Response Spectrum [IS 1893 (Part 1): 2002, Clause 6.4.5]**

- The **design response spectrum** is a smooth response spectrum specifying the level of seismic resistance required for a design.
- Seismic analysis requires that the design spectrum be specified.
- IS 1893 (Part 1): 2002 stipulates a design acceleration spectrum or base shear coefficients as a function of natural period.
- These coefficients are ordinates of the acceleration spectrum divided by acceleration due to gravity.
- This relationship works well in SDOF systems.
- The spectral ordinates are used for the computation of inertia forces.
- **Fig. 4.2** relates to the proposed 5 percent damping for rocky or hard soils sites and **Table 4.4** gives the multiplying factors for obtaining spectral values for various other damping (note that the multiplication is not to be done for zero period acceleration).
- The design spectrum ordinates are independent of the amounts of damping (multiplication factor of 1.0) and their variations from one material or one structural solution to another.

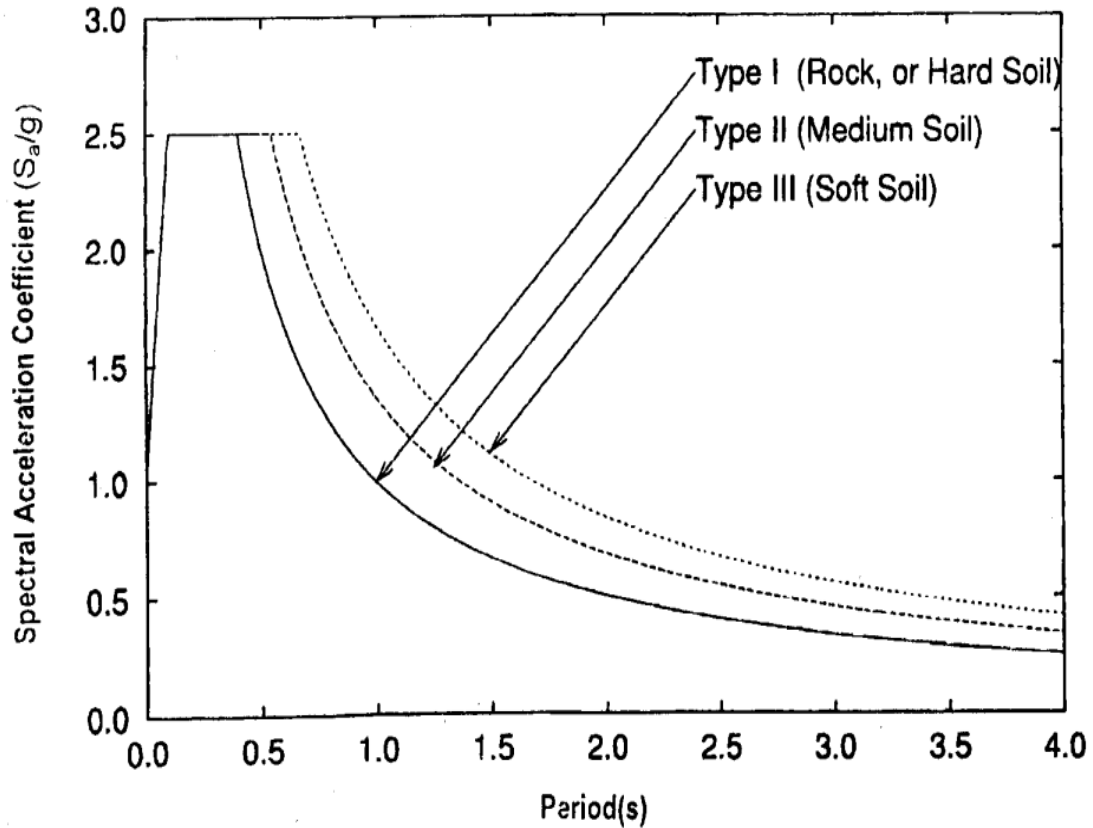


Fig. 4.2. Response Spectra for rock and soil sites for 5 % damping

**Table 4.4 Multiplying Factors for Obtaining Values for damping (other than 5%)**

Damping, percent	0	2	5	7	10	15	20	25	30
Factors	3.20	1.40	1.00	0.90	0.80	0.70	0.60	0.55	0.50

#### 4.5. SEISMIC BASE SHEAR [IS 1893 (Part 1): 2002, Clause 7.5.3]

- The total design lateral force or design seismic base shear ( $V_B$ ) along any principal direction is determined by

$$V_B = A_h W \quad \dots \text{Equation 4.1}$$

Where  $A_h$  is the design horizontal acceleration spectrum value using the fundamental natural period,  $T$ , in the considered direction of vibration and  $W$  is the seismic weight of the building.

- The design horizontal seismic coefficient  $A_h$  for a structure is determined by the expression [IS 1893 (Part 1): 2002, Clause 6.4.2]

$$A_h = \frac{ZIS_a}{2Rg} \quad \dots \text{Equation 4.2}$$

- For any structure with  $T \leq 0.1s$ , the value of  $A_h$  will not be taken less than  $Z/2$  whatever be the value of  $I/R$ .
- In **Eqn (4.2)**, **Z** is the zone factor as discussed previously, for the maximum considered earthquake (MCE).
- The factor **2** in the denominator is used so as to reduce the maximum considered earthquake (MCE) zone factor to the factor for design basis earthquake (DBE).
- **I** is the importance factor as discussed previously and depends upon the functional use of the structure, the hazardous consequences of its failure, post-earthquake functional needs, historical value, or economic importance.
- **R** is the response reduction factor as discussed previously and depends on the perceived seismic damage performance of the structure, characterized by ductile or brittle deformations. This factor is used to decide what other building materials are used, the type of construction and the type of lateral bracing system.
- As given by **fig. 4.2**,  $S_a/g$  is the response acceleration as defined previously for 5% damping based on appropriate natural periods. The curves of **fig. 4.2** represent free-fluid ground motion.
- For other damping values of the structure, multiplying factors given in **Table 4.4** should be used.
- Response acceleration coefficient is given in **Table 4.5**.



**Table 4.5 Response acceleration coefficient ( $S_a/g$ )**

<i>For rocky, or hard soil sites</i>	
$\frac{S_a}{g} = \begin{cases} 1 + 15 T; & 0.00 \leq T \leq 0.10 \\ 2.50 & 0.10 \leq T \leq 0.40 \\ 1.00/T & 0.40 \leq T \leq 4.00 \end{cases}$	
<i>For medium soil sites</i>	
$\frac{S_a}{g} = \begin{cases} 1 + 15 T; & 0.00 \leq T \leq 0.10 \\ 2.50 & 0.10 \leq T \leq 0.55 \\ 1.36/T & 0.55 \leq T \leq 4.00 \end{cases}$	
<i>For soft soil sites</i>	
$\frac{S_a}{g} = \begin{cases} 1 + 15 T; & 0.00 \leq T \leq 0.10 \\ 2.50 & 0.10 \leq T \leq 0.67 \\ 1.67/T & 0.67 \leq T \leq 4.00 \end{cases}$	

#### **4.6. SEISMIC WEIGHT [IS 1893 (Part 1): 2002, Clause 7.4]**

- The seismic weight of the whole building is the sum of the seismic weights of all the floors.
- The seismic weight of each floor is its full dead load (DL) plus the appropriate amount of imposed dead (IL), the latter being that part of the imposed loads that may reasonably be expected to be attached to the structure at the time of earthquake shaking.
- It includes the weight of permanent and movable partitions, permanent equipment, a part of the live load etc.
- While computing the seismic weight of each floor, the weight of columns and walls in any storey should be equally distributed to the floors above and below the storey.
- Any weight supported in between storeys should be distributed to the floors above and below in inverse proportion to its distance from the floors [IS 1893 (Part 1): 2002, Clause 7.3].

- As per IS 1893 (Part 1), the percentage of imposed load as given in the **Table 4.6** should be used. For calculating the design seismic forces of the structure, the imposed load on the roof need not be considered.
- A reduction in IL is recommended for the following reasons.
  - All the floors may not be occupied during earthquake
  - A part of earthquake energy may get absorbed by non-rigid mountings of IL.

**Table 4.6 Percentage of imposed load to be considered in seismic weight calculation**

<b>Imposed Uniformly Distributed Floor Loads (KN/m<sup>2</sup>)</b>	<b>Percentage of Imposed Load</b>
Upto and including 3.0	25
Above 3.0	50

#### **4.7. DISTRIBUTION OF DESIGN FORCE [IS 1893 (Part 1): 2002, Clause 7.7]**

- Buildings and their elements should be designed and constructed to resist the effects of design lateral force.
- The design lateral force is first computed for the building as a whole and then distributed to the various floor levels.
- The overall design seismic force thus obtained at each floor level is then distributed to individual lateral load resisting elements depending on the floor diaphragm action.

##### **4.7.1 Seismic Coefficient Method**

- **This is also known as equivalent lateral force method or equivalent static method or static method.**
- This method is the simplest one - it requires less computational effort and is based on formulae given in the code of practice.
- First, the design base shear is computed for the whole building and it is then distributed along the height of the building. The lateral forces at each floor level thus obtained are distributed to individual lateral load resisting elements.

- **Vertical distribution of base shear to different floor levels [IS 1893 (Part 1): 2002, Clause 7.7.1]**

- The design base shear ( $V_B$ ) is distributed along the height of the building as per the following expression.

$$Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^n W_j h_j^2}$$

Where  $Q_i$  is the design lateral force at floor  $i$ ,  $W_i$  is the seismic weight of the floor  $i$ ,  $h_i$  is the height of floor  $i$  measured from the base, and  $n$  is the number of storeys in the building i.e, the number of levels at which the masses are located.

- **Distribution of horizontal design lateral force to different lateral force resisting elements [IS 1893 (Part 1): 2002, Clause 7.7.2]**

- In the case of buildings in which floors are capable of providing rigid horizontal diaphragm action, the total shear in any horizontal plane is distributed to the various vertical elements of the lateral force-resisting system, assuming the floors to be infinitely rigid in the horizontal plane.
- For buildings in which floor diaphragms cannot be treated as infinitely rigid in their own plane, the lateral shear at each floor is distributed to the vertical elements resisting the lateral forces, accounting for the in-plane flexibility of the diaphragms.

## 4.8 PROBLEMS

### PROBLEM 4.1

A 20 storey R.C framed building has plan dimensions 15 m × 30 m. Height of the building is 70 m. Estimate its fundamental period of vibration if the building is (a) unbraced, and (ii) braced with infilled brick masonry walls.

*Solution*

The fundamental period of vibration can be estimated using empirical equations as per IS:1893-part 1:

(a) *Unbraced Building*

$$\begin{aligned} T &= 0.075 H^{0.75} \\ &= 0.075 \times 70^{0.75} = 1.82 \text{ sec} \end{aligned}$$

(b) *Braced Building*

$$T = \frac{0.09H}{\sqrt{D}}$$

$$\text{In short direction } T = \frac{0.09 \times 70}{\sqrt{15}} = 1.63 \text{ sec}$$

$$\text{In long direction } T = \frac{0.09 \times 70}{\sqrt{30}} = 1.15 \text{ sec}$$

### PROBLEM 4.2

A four storeyed building has an elevation shown in Fig. 20.10 and is located in seismic zone IV. Determine the lateral forces and storey shears on an inner frame due to earthquake using the following data:

Bay width = 6 m centre to centre (c/c);  
Frame spacing = 5 m c/c  
Height of ground floor = 4 m,

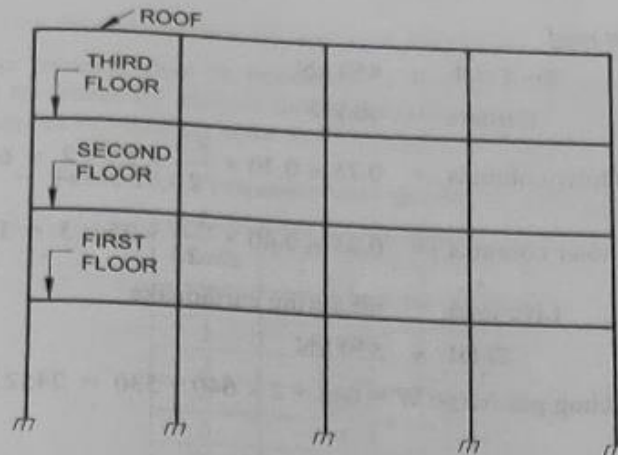


Fig. 20.10

Height of other floors	= 3.5 m
Floor thickness including finish	= 15 cm
Outer columns	= 25 cm × 30 cm, 2 No.;
Inner columns	= 25 cm × 40 cm 3 No.
Girders below floor slab	= 25 cm × 40 cm
Live load	= 3 kN/m <sup>2</sup>

There are no walls in this frame. Its foundation rests on hard soil.

*Solution*

(i) *Weight lumped at first floor*

At any floor, half of the weight of walls and columns below it and half of that above it are lumped at this level along with the weight of floor and girders.

$$\text{Density of concrete} = 25 \text{ kN/m}^3$$

$$\text{Floor slab} = 0.15 \times 24 \times 5 \times 25 = 450 \text{ kN}$$

$$\text{Girder} = 0.25 \times 0.40 \times 24 \times 25 = 60 \text{ kN}$$

$$\text{Outer columns} = 0.25 \times 0.30 \times \left( \frac{4 + 3.5}{2} \right) \times 25 \times 2 = 14.1 \text{ kN}$$

$$\text{Inner columns} = 0.25 \times 0.40 \times \left( \frac{4 + 3.5}{2} \right) \times 25 \times 3 = 28.125 \text{ kN}$$

$$25\% \text{ of live load} = 0.25 \times 3 \times 24 \times 5 = 90 \text{ kN}$$

$$\text{Total} \approx 642 \text{ kN}$$

(ii) *Weight lumped at second and third floors*

$$\text{Floor slab} = 450 \text{ kN}$$

$$\text{Girder} = 60 \text{ kN}$$

$$\text{Outer columns} = 0.25 \times 0.30 \times 3.5 \times 25 \times 2 = 13.10 \text{ kN}$$

$$\text{Inner columns} = 0.25 \times 0.40 \times 3.5 \times 25 \times 3 = 26.25 \text{ kN}$$

$$25\% \text{ of live load} = 90 \text{ kN}$$

$$\text{Total} = 640 \text{ kN}$$

(iii) Weight lumped at roof

Roof slab = 450 kN

Girders = 60 kN

Outer columns =  $0.25 \times 0.30 \times \frac{3.5}{2} \times 25 \times 2 = 6.60 \text{ kN}$

Inner columns =  $0.25 \times 0.40 \times \frac{3.5}{2} \times 25 \times 3 = 13.125 \text{ kN}$

Live load = nil during earthquake

Total  $\approx 530 \text{ kN}$

Total weight of building per frame  $W = 642 + 2 \times 640 + 530 = 2452 \text{ kN}$

(iv) Base shear

$$V_b = \alpha_h W D = \left( \frac{Z}{2} \right) \left( \frac{I}{R} \right) \left( \frac{S_a}{g} \right) W D$$

Zone factor  $Z = 0.24$

Importance factor  $I = 1$

Response reduction factor  $R = 4$

Fundamental period  $T = 0.075 H^{0.75} = 0.075 \times 14.5^{0.75} = 0.56 \text{ second}$

For  $T = 0.56 \text{ sec}$ , 5% damping and hard soil site,  $\left( \frac{S_a}{g} \right) = 2.0$ ,  $D = 1$

$$\therefore \text{Base shear } V_b = \left( \frac{0.24}{2} \right) \left( \frac{1}{4} \right) \times 2 \times 2452 = 147 \text{ kN}$$

(v) Lateral forces

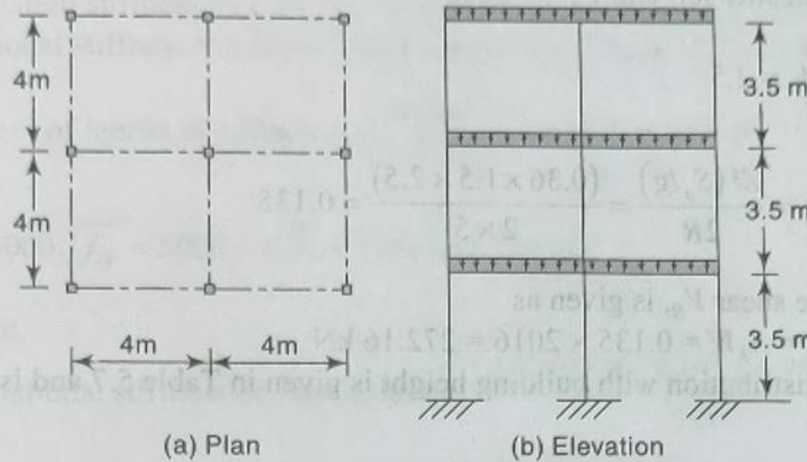
Distribution of earthquake force on different floors of the frame under consideration is parabolic and is given by Eq. 20.8. The calculations are shown in Table 20.5. Storey shears  $V_i$  are computed by adding lateral forces from the top floor.

Table 20.5 Lateral forces and storey shears

Floor	$W_i \text{ kN}$	$h_i \text{ m}$	$W_i h_i^2$	$F_i \text{ kN}$	$V_i \text{ kN}$
R	530	14.5	111432	69.66	69.66
3	640	11.0	77440	48.42	118.08
2	640	7.5	36000	22.50	140.58
1	642	4.0	10272	6.42	147.00
			$\Sigma 235144$		

### **PROBLEM 4.3**

The plan and elevation of a three-storey RCC school building is shown in Fig. 5.28. The building is located in seismic zone V. The type of soil encountered is medium stiff and it is proposed to design the building with a special moment-resisting frame. The intensity of DL is  $10 \text{ kN/m}^2$  and the floors are to cater to an IL of  $3 \text{ kN/m}^2$ . Determine the design seismic loads on the structure by static analysis.



**Fig. 5.28** Building configuration



### Solution

Design parameters:

For seismic zone V, zone factor,  $Z = 0.36$

Importance factor,  $I = 1.5$

Response reduction factor  $R = 5$

Seismic weight:

Floor area  $= 8 \times 8 = 64 \text{ m}^2$

For live load up to and including  $3 \text{ kN/m}^2$ ,

percentage of live load to be considered  $= 25\%$ .

The total seismic weight on the floors is

$$W = \sum W_i$$

where  $W_i$  is sum of loads from all the floors which includes DLs and appropriate percentage of live loads.

Seismic weight contribution from one floor  $= 64 \times (10 + 0.25 \times 3) = 688 \text{ kN}$

Load from roof  $= 64 \times 10 = 640 \text{ kN}$

Hence, the total seismic weight of the structure  $= 2 \times 688 + 640 = 2016 \text{ kN}$

Fundamental natural period of vibration, is given as

$$T_a = \frac{0.09h}{\sqrt{d}}$$

where  $h$  is the height of the building in metres and  $d$  is the base dimension in metres at plinth level along the direction of the lateral load.

$$T_a = \frac{0.09 \times 10.5}{\sqrt{8}} = 0.334 \text{ s}$$

Since the building is symmetrical in plan, the fundamental natural period of vibration will be the same in both the directions.

For medium stiff soil and  $T_a = 0.334 \text{ s}$

$$\frac{S_a}{g} = 2.5$$

$$A_h = \frac{ZI(S_a/g)}{2R} = \frac{(0.36 \times 1.5 \times 2.5)}{2 \times 5} = 0.135$$

Design base shear  $V_B$ , is given as

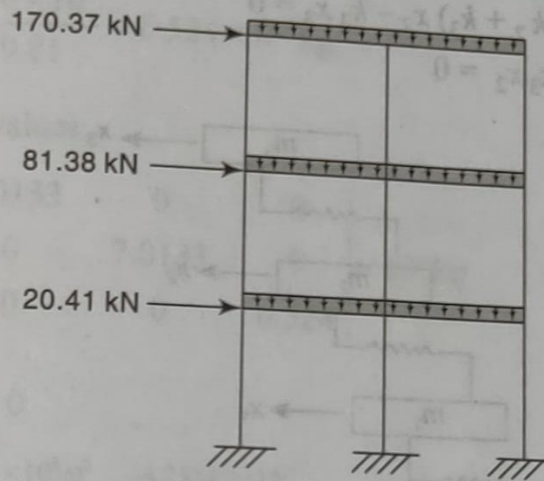
$$V_B = A_h W = 0.135 \times 2016 = 272.16 \text{ kN}$$

The force distribution with building height is given in Table 5.7 and is shown in Fig. 5.29.



**Table 5.7** Lateral load distribution with height

Storey level	$W_i$ (kN)	$h_i$ (m)	$W_i h_i^2$		Lateral force $V_i$ at $i^{\text{th}}$ level for EL in x and y directions (kN)
3	640	10.5	70560		
2	688	7	33712	0.626	170.37
1	688	3.5	8428	0.299	81.38
			$\Sigma W_i h_i^2 = 112700$	0.0748	20.41
					$\Sigma V_i = 272.16$



**Fig. 5.29** Design seismic forces by static analysis

#### PROBLEM 4.4

A 10-storey OMRF building has plan dimensions as shown in Fig. 5.33. The storey height is 3.0 m. The DL per unit area of the floor, consisting of the floor slab, finishes, etc., is  $4 \text{ kN/m}^2$ . Weight of the partitions on the floor can be assumed to be  $2 \text{ kN/m}^2$ . The intensity of live load on each floor is  $3 \text{ kN/m}^2$  and on the roof is  $1.5 \text{ kN/m}^2$ . The soil below the foundation is hard and the building is located in Delhi. Determine the seismic forces and shears at different floor levels.

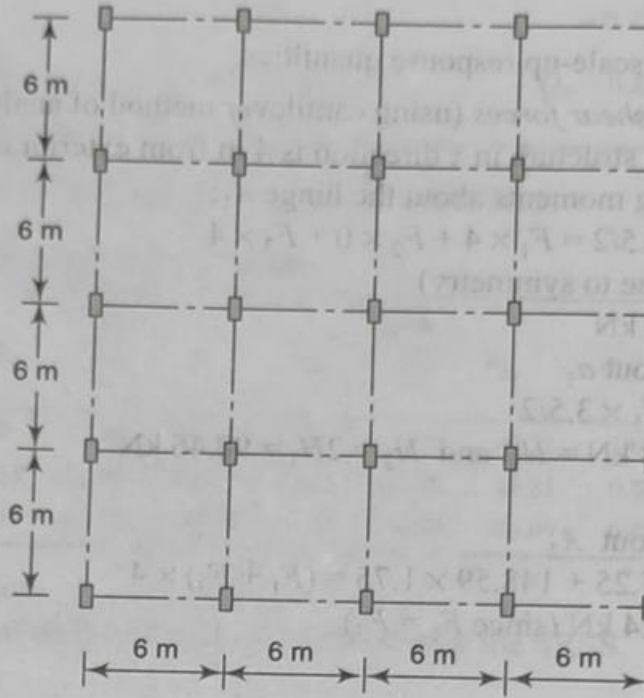


Fig. 5.33 Plan

### Solution

Design parameters:

For Delhi (Zone IV), zone factor  $Z = 0.24$

Importance factor  $I = 1.0$

Response reduction factor  $R = 3.0$

Seismic weight:

Floor area  $= 24 \times 24 = 576 \text{ m}^2$

Dead load  $= 4 \text{ kN/m}^2$

Weight of partitions  $= 2 \text{ kN/m}^2$

For live load upto and including  $3 \text{ kN/m}^2$ ,

percentage of live load to be considered  $= 25\%$

Total seismic weight on the floors,

$$W = \sum W_i$$

where  $\sum W_i$  is the sum of loads from all the floors, which includes dead loads and appropriate percentage of live loads.

Effective weight at each floor except the roof  $= 4.0 + 2.0 + 0.25 \times 3 = 6.75 \text{ kN/m}^2$ , and at the roof  $= 4.0 \text{ kN/m}^2$ .

Weight of the beams at each floor and the roof  $= 0.3 \times 0.6 \times 240 \times 25 = 1080 \text{ kN}$

Weight of the columns at each floor  $= 0.3 \times 0.6 \times 2.4 \times 25 \times 25 = 270 \text{ kN}$

Weight of the column at the roof  $= \frac{1}{2} \times 270 = 135 \text{ kN}$

Total plan area of the building is  $24 \text{ m} \times 24 \text{ m} = 576 \text{ m}^2$

Equivalent load at roof level  $= 4 \times 576 + 1080 + 135 = 3519 \text{ kN}$

Equivalent load at each floor  $= 6.75 \times 576 + 1080 + 270 = 5238 \text{ kN}$

Seismic weight of the building  $W = 3519 + 5238 \times 9 = 50661 \text{ kN}$

Base shear:

Fundamental natural period of vibration of a moment-resisting frame without infill

$$T_a = 0.075h^{0.75} = 0.075(30)^{0.75} = 0.96 \text{ s}$$

Average response acceleration coefficient  $S_a/g$  for 5% damping and type I soil is 1.04.

Design horizontal seismic coefficient,

$$A_h = \frac{ZI(S_a/g)}{2R} = \frac{0.24 \times 1.0 \times 1.04}{2 \times 3} = 0.0416$$

Base shear  $V_B = A_h W = 0.0416 \times 50661 = 2107.5 \text{ kN}$

Lateral load and shear force at various floor levels

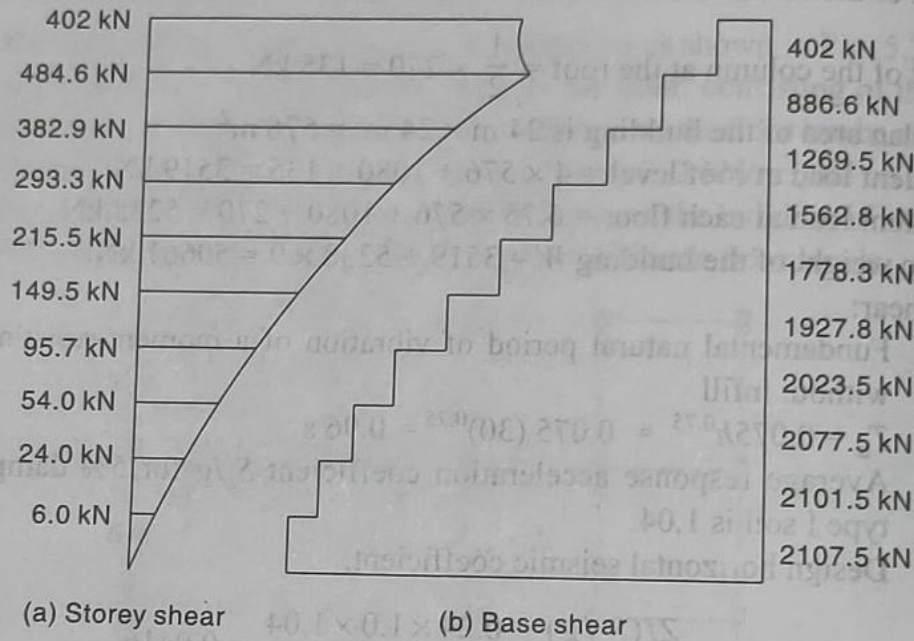
$$\text{Design lateral force at floor } i, Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^n W_j h_j^2}$$

The calculation of design lateral forces at each floor level is shown in Table 5.12.

**Table 5.12** Lateral loads and shear forces at different floors levels

Mass No.	$W_i$ (kN)	$h_i$ (m)	$W_i h_i^2$ (kN-m <sup>2</sup> )	$\frac{W_i h_i^2}{\sum_{i=1}^n W_i h_i^2}$	$Q_i$ (kN)	$V_i$ (kN)
1	3519	30.0	3167100	0.1907	402.0	402.0
2	5238	27.0	3818502	0.2299	484.6	886.6
3	5238	24.0	3017088	0.1817	382.9	1269.5
4	5238	21.0	2309958	0.1391	293.3	1562.8
5	5238	18.0	1697112	0.1022	215.5	1778.3
6	5238	15.0	1178550	0.0709	149.5	1927.8
7	5238	12.0	754272	0.0454	95.7	2023.5
8	5238	9.0	424278	0.0255	54.0	2077.5
9	5238	6.0	188568	0.0114	24.0	2101.5
10	5238	3.0	47142	0.0028	6.0	2107.5
$\sum W_i h_i^2 = 16602570$						

The design seismic forces at different floor levels are shown in Fig. 5.34.

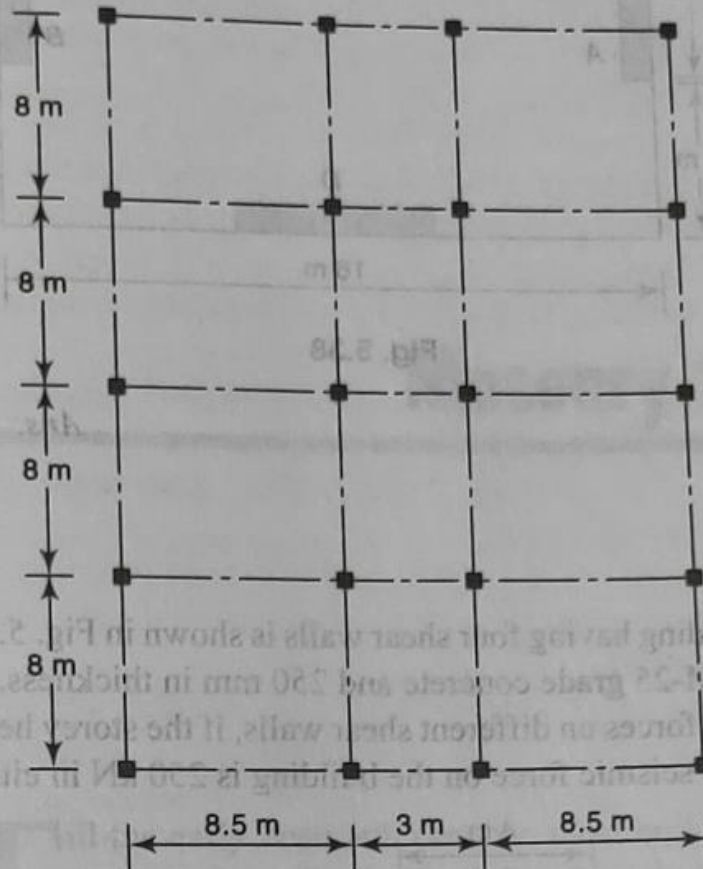


**Fig. 5.34** Design seismic forces



### PRACTICE QUESTION

Plan of a five-storey building is shown in Fig. 5.37. Dead load including self weight of slab, finishes, etc. can be assumed as  $3 \text{ kN/m}^2$  and live load as  $4 \text{ kN/m}^2$  on each floor and as  $1.5 \text{ kN/m}^2$  on the roof. Weight of partitions is  $2 \text{ kN/m}^2$ . Determine the lateral forces and shears at different storey levels.



**Fig. 5.37** Plan

# **LECTURE – 5**

**SIGNIFICANCE OF DUCTILITY**

**DUCTILITY OF BEAM**

**DESIGN AND DETAILING FOR DUCTILITY**

**SIMPLE PROBLEMS BASED ON ABOVE CONCEPT AS PER IS 13920**

### 21.3 SIGNIFICANCE OF DUCTILITY

When a ductile structure is subjected to overloading it will tend to deform inelastically and in doing so, will redistribute the excess load to elastic parts of the structure. This concept can be utilized in several ways :

- (1) If a structure is ductile, it can be expected to adapt to unexpected overloads, load reversals, impact and structural movements due to foundation settlement and volume changes. These items are generally ignored in the analysis and design but are assumed to have been taken care of by the presence of some ductility in the structure.
- (2) If a structure is ductile, its occupants will have sufficient warning of the impending failure thus reducing the probability of loss of life in the event of collapse.
- (3) The limit state design procedure assumes that all the critical sections in the structure will reach their maximum capacities at design load for the structure. For this to occur, all joints and splices must be able to withstand forces and deformations corresponding to yielding of the reinforcement.

#### Ductility

Member or structural ductility is defined as the ratio of absolute maximum deformation to the corresponding yield deformation. However, ductility has no precise meaning until the method of measuring the deformation has been defined. This can be defined with respect to *strains, rotations, curvatures or deflections*. Strain based ductility definition depends almost exclusively on the material, while rotation or curvature based ductility definition also includes the effects of shape and size of the cross-section. When the definition is applied to deflections, the entire configuration of structure and loading is also taken into account. It should be noted that no definition claims any special merit over the others.

In the typical force-deformation relation plotted in Fig. 21.1, the force may be load, moment or stress, while the deformation could be elongation, curvature, rotation or strain.  $\Delta_y$  is the yield deformation corresponding to yielding of the reinforcement in a cross-section or to a major deviation from the linear force-deformation curve for a member or structure.  $\Delta_u$  is the ultimate deformation beyond which the force-deformation curve has a negative slope. The ductility  $\mu$  is defined by the equation:

$$\mu = \frac{\Delta_u}{\Delta_y} \text{ with respect to displacement} \quad (21.1a)$$

$$= \frac{\phi_u}{\phi_y} \text{ with respect to curvature} \quad (21.1b)$$

$$= \frac{\theta_u}{\theta_y} \text{ with respect to rotation} \quad (21.1c)$$

#### Ductility requirements

The assignment of structural ductility factor to a structure has to be consistent with the capability of the associated detailing of reinforcement as well as material strains within the structure. It would depend upon:

- (a) Plastic hinge mechanisms that may develop within the structure,
- (b) Material strain demands within the inelastic zones,

- (c) Material strain capabilities, and  
 (d) Inelastic displacement profile of the structure, usually base shear – roof displacement plot.

## 21.4 DUCTILITY OF BEAMS

The ductility of reinforced concrete beams may be defined in terms of the behaviour of individual cross-section or the behaviour of entire beam. The former definition is more widely used because the behaviour of cross-section is much better defined and it is easier to compute.

Let us derive expression for the curvature ductility of a beam. With reference to Fig. 21.5a, the yield curvature of a singly reinforced beam can be computed using the elastic theory, that is,

$$\phi_y = \frac{\epsilon_y}{d - Nd} \quad (21.2a)$$

where,  
 $\epsilon_y$  = yield strain of the tensile reinforcement =  $\sigma_y/E_s$   
 $d$  = effective depth  
 $Nd$  = depth of neutral axis computed using elastic theory

$$N = -mp + \sqrt{m^2 p^2 + 2mp} \quad (21.2b)$$

$m$  = modular ratio  $\approx 280/3 \sigma_{cbc}$

$p$  = tension steel ratio =  $A_t/bd$

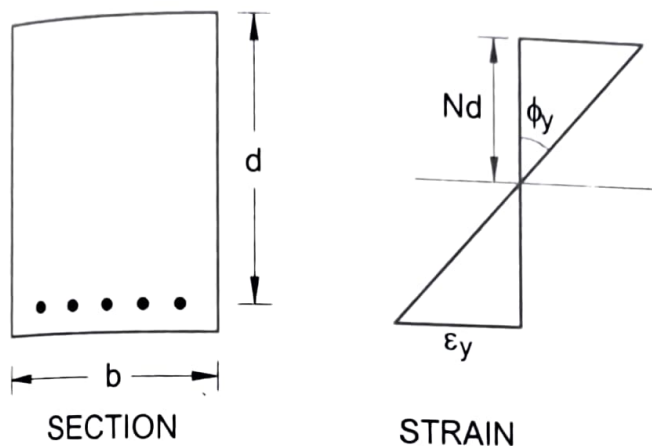


Fig. 21.5a Yield curvature

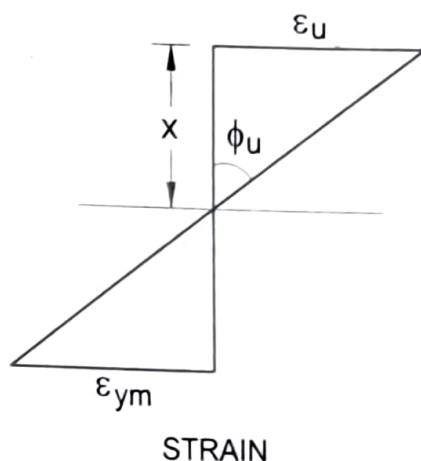


Fig. 21.5b Ultimate curvature

Similarly with reference to Fig. 21.5b, the ultimate curvature can be computed as,

$$\phi_u = \frac{\epsilon_u}{x} \quad (21.2c)$$

where,  $\epsilon_u$  = ultimate strain at crushing of concrete = 0.0035

$$x = \frac{0.87 \sigma_y A_t}{0.36 \sigma_{ck} b} = \frac{0.87 \sigma_y p d}{0.36 \sigma_{ck}} \quad (21.2d)$$

$$\leq x_m$$



Substituting Eqs. 21.2a and 21.2c in Eq. 21.1b gives,

$$\mu = \frac{\epsilon_u}{\epsilon_y} \left( \frac{d - Nd}{x} \right) \quad (21.3a)$$

$$\text{or,} \quad \mu = \frac{\epsilon_u}{\sigma_y / E_s} \left[ \frac{1 + mp - \sqrt{m^2 p^2 + 2mp}}{x/d} \right] \quad (21.3b)$$

In the case of a doubly reinforced beam, a similar expression for ductility factor can be derived. The addition of compression reinforcement to a beam has relatively little effect on its yield curvature. It does, however, greatly increase the ultimate curvature. The depth of neutral axis at collapse can be determined from the expression :

$$0.36 \sigma_{ck} bx + \sigma'_y A_{sc} = 0.87 \sigma_y A_t$$

$$\text{or,} \quad \frac{x}{d} = \left[ 0.87p - \frac{\sigma'_y}{\sigma_y} p_c \right] \frac{\sigma_y}{0.36 \sigma_{ck}} \quad (21.4a)$$

where,  $\sigma'_y$  = stress in the compression reinforcement

$$p_c = A_{sc}/bd$$

$$\text{If } \sigma'_y = 0.87 \sigma_y$$

Eq. 21.4a becomes

$$\frac{x}{d} = (p - p_c) \frac{0.87 \sigma_y}{0.36 \sigma_{ck}} \quad (21.4b)$$

$$\leq x_m$$

Fig. 21.5 also gives  $\frac{x}{d - x} = \frac{\epsilon_u}{\epsilon_{ym}}$

$$\text{or,} \quad \frac{x}{d} = \frac{\epsilon_u}{\epsilon_u + \epsilon_{ym}} \quad (21.4c)$$

where,

$$\epsilon_{ym} = \text{maximum strain in tensile steel} = \mu_s \epsilon_u$$

$$\mu_s = \text{strain ductility in steel}$$

Eq. 21.4b can be rewritten as

$$(p - p_c) \leq \left[ \frac{\epsilon_u}{\epsilon_u + \mu_s \epsilon_y} \right] \frac{0.36 \sigma_{ck}}{0.87 \sigma_y} \quad (21.4d)$$

Thus ductility can be obtained by using either Eqs. 21.3b and 21.4b, or Eqs. 21.3a and 21.4c. Values of  $\mu$  calculated using Eq. 21.3b and Eq. 21.4b are presented in Fig. 21.6 for Fe 415 grade steel.

## Variables affecting the ductility

### (1) Tension steel ratio $p$

As shown in Fig. 21.6 the ductility of a beam cross-section increases as the steel ratio  $p$  or  $(p - p_c)$  decreases. If excessive reinforcement is provided, the concrete will crush before the steel yields, leading to a brittle failure corresponding to  $\mu = 1.0$ . In other words, a beam should be designed as under reinforced.

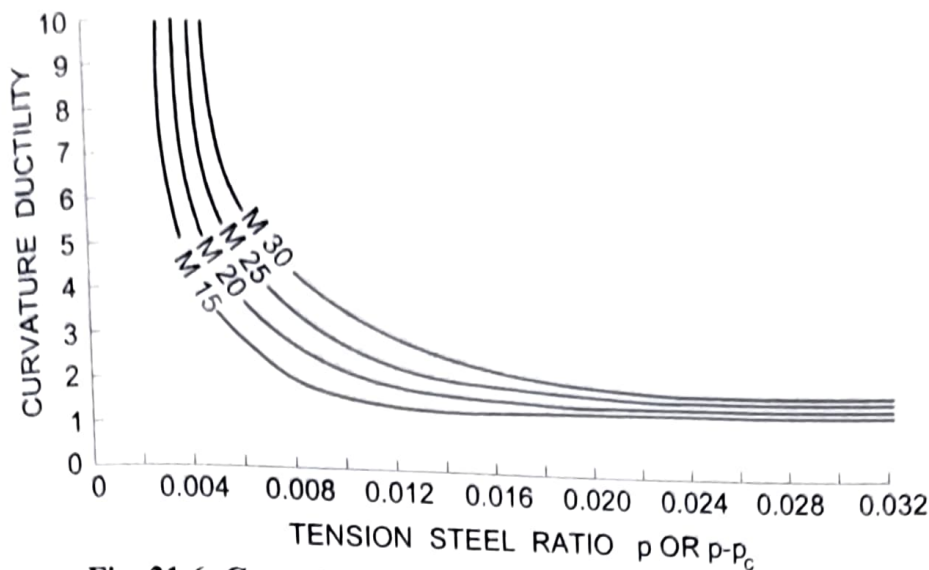


Fig. 21.6 Curvature ductility in beams with Fe 415 steel

The ductility is directly affected by the values  $\epsilon_u$ ,  $\sigma_{ck}$  and  $\sigma_y$ . The ultimate strain  $\epsilon_u$  is a function of a number of variables such as the characteristic strength of concrete, rate of loading and strengthening effect of stirrups. The Code recommends a value of 0.0035 for  $\epsilon_u$ . It can be seen in Fig. 21.6 that ductility increases with the increase in characteristic strength of concrete. Eq. 21.3 shows that ductility decreases with the increase in characteristic strength of steel. In fact, ductility is inversely proportional to the square of  $\sigma_y$ . It suggests that Fe 415 grade steel is more desirable from the ductility point of view as compared with Fe 500 grade high strength steel.

### (2) Compression steel ratio $p$

Eq. 21.4b suggests that  $(p - p_c)$  is an important parameter defining the ductility ratio. Figure 21.6 shows that ductility increases with the decrease in  $(p - p_c)$  value, that is, ductility increases with the increase in compression steel.

### (3) Shape of cross-section

The presence of an enlarged compression flange in a T-beam reduces the depth of the compression zone at collapse and thus increases the ductility. If neutral axis falls in the flange, then ductility can be calculated using Fig. 21.6.

### (4) Lateral reinforcement

Lateral reinforcement tends to improve ductility by preventing premature shear failures, restraining the compression reinforcement against buckling and by confining the compression zone, thus increasing deformation capability of a reinforced concrete beam.

## 21.5 DESIGN FOR DUCTILITY

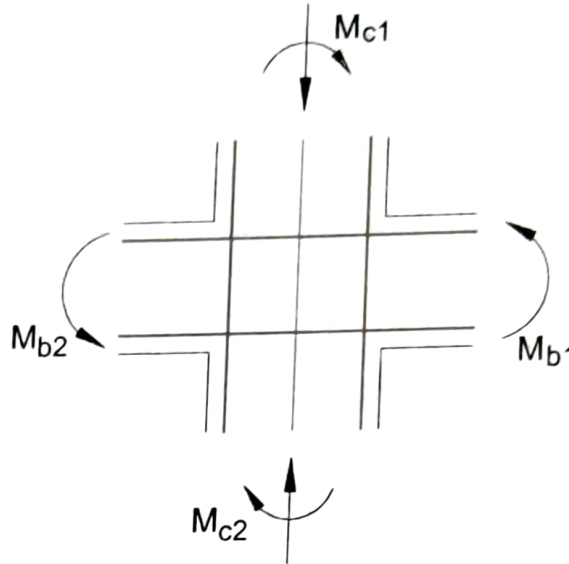
Selection of cross-sections that will have adequate strength is rather easy. But it is

much more difficult to achieve the desired strength as well as ductility. To ensure sufficient ductility, the designer should pay attention to detailing of reinforcement, bar cut-offs, splicing and joint details. Sufficient amount of ductility can be ensured by following certain simple design details such as :

- (1) The structural layout should be simple and regular avoiding offsets of beams to columns, or offsets of columns from floor to floor. Changes in stiffness should be gradual from floor to floor.
- (2) The amount of tensile reinforcement in beams should be restricted and more compression reinforcement should be provided. The latter should be enclosed by stirrups to prevent it from buckling.
- (3) Beams and columns in a reinforced concrete frame should be designed in such a manner that inelasticity is confined to beams only and the columns should remain elastic. To ensure this, sum of the moment capacities of the columns for the design axial loads at a beam-column joint should be greater than the moment capacities of the beams along each principal plane.

$$\Sigma M_{\text{column}} > 1.2 \Sigma M_{\text{beam}} \quad (21.5)$$

The flexural resistances be summed such that the column moments oppose the beam moments as shown in Fig. 21.7.



**Fig. 21.7 Weak girder-strong column requirement**

- (4) The shear reinforcement should be adequate to ensure that the strength in shear exceeds the strength in flexure and thus, prevent a non-ductile shear failure before the fully reversible flexural strength of a member has been developed.

Clause 6.3.3 of IS: 13920-1993 requires that the shear resistance shall be the maximum of the:

- (a) Calculated factored shear force as per analysis, and
- (b) shear force due to formation of plastic hinges at both ends plus the factored gravity loads on the span.

- (i) For sway to right (Fig. 21.8)

$$V_a = 1.4 \left[ \frac{M_u^{As} + M_u^{Bh}}{L_c} \right] + 0.5wL_c$$

$$V_b = 1.4 \left[ \frac{M_u^{As} + M_u^{Bs}}{L_c} \right] - 0.5wL_c \quad (21.6a)$$

(ii) For sway to left

$$V_a = 1.4 \left[ \frac{M_u^{Ah} + M_u^{Bs}}{L_c} \right] - 0.5wL_c$$

$$V_b = 1.4 \left[ \frac{M_u^{Ah} + M_u^{Bs}}{L_c} \right] + 0.5wL_c \quad (21.6b)$$

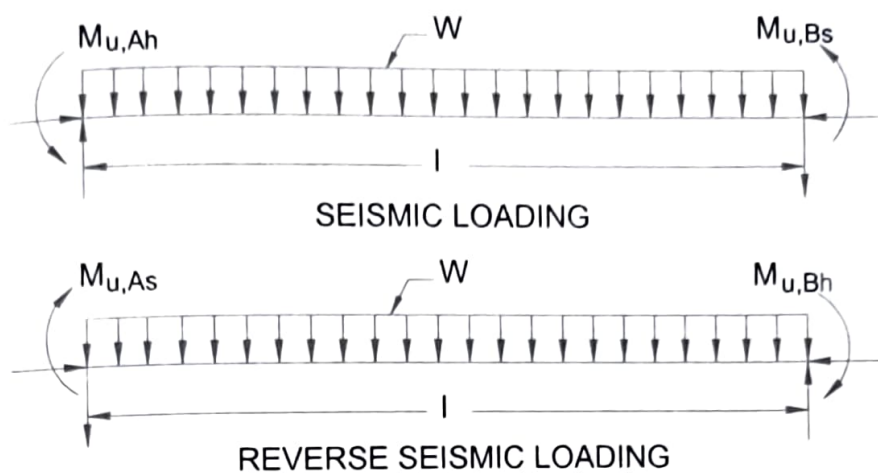
where,  
 $M_u$  = hogging or sagging moment capacity at the left end of the beam = 1.4 times the yield moment capacity at each end of the beam

$M_u^{As}$  = sagging moment capacity at the left end of the beam

$M_u^{bh}$  = hogging moment capacity at the right end of the beam

$w$  = factored gravity load = 1.2 (DL + LL)

$L_c$  = clear span of beam



**Fig. 21.8 Reversal of shear due to seismic loading**

The resistance  $M_p$  corresponds to the moment of resistance of the beam section on either side of the joint. It is assumed that the ratio of the actual ultimate tensile stress to the actual tensile yield strength of the steel is not less than 1.25. Use of longitudinal reinforcement with yield strength substantially higher than that assumed in the design will lead to higher shear and bond stresses at the time of development of yield moment. This may lead to unexpected brittle failures and should be avoided. It is known that the length of the yield region is related to the relative magnitudes of the ultimate and yield strengths. The larger is the ratio of the ultimate to yield moment, the longer is the yield region. The factor 1.4 is equal to 1.25 times the yield strength of steel divided by 0.87 (that is  $1.25/0.87 = 1.43 \approx 1.40$ ).

The design shear at each end A and B shall be the absolute maximum of the corresponding two values of  $V_a$  and  $V_b$ .



(5) The design shear force for columns will be the maximum of:

(a) Calculated factored shear force, and

(b) A factored shear force given by:

$$V_u = 1.4 \left[ \frac{M_u^{As} + M_u^{Bh}}{h_c} \right] \quad (21.6c)$$

(6) Closed stirrups or spirals should be used to confine the concrete at sections of maximum moment to increase the ductility of members. Such sections include upper and lower ends of columns, and within beam-column joints which do not have beams on all sides. If axial load exceeds 0.4 times the balanced axial load, a spiral column is preferred.

(7) Splices and bar anchorages must be adequate to prevent bond failures.

(8) The reversal of stresses in beams and columns due to reversal of direction of earthquake force must be taken into account in the design by providing appropriate reinforcement.

(9) Beam-column connections should be made monolithic.

## 21.6 DETAILING FOR DUCTILITY

The following recommendations are based on the provisions of IS : 4326-1993, IS : 13920-1993 and ACI 318-2008 and lessons learnt from the failure of concrete structures during past earthquakes.

### (A) Girders

A girder should satisfy the following requirements:

(1) The factored axial stress on these members under earthquake loading should not exceed  $0.1\sigma_{ck}$ .

(2) The width to depth ratio of the member is more than 0.3.

(3) The depth of the member is not more than 1/4th of the clear span.

### Longitudinal Reinforcement

(1) At any section of a flexural member and for the top as well as for the bottom reinforcement:

(i) the reinforcement ratio  $p$  should each be greater than

$$\text{IS : 13920-1993} \quad p > \frac{0.24\sqrt{\sigma_{ck}}}{\sigma_y} \quad (21.7a)$$

$$\text{ACI 318-2008} \quad p > \frac{0.25\sqrt{\sigma'_{ck}}}{\sigma_y} \quad \text{but not less than} \quad p > \frac{1.4}{\sigma_y} \quad (21.7b)$$

where

$\sigma'_{ck}$  = cylinder strength of concrete

$p$  =  $\frac{A_s}{bd}$  for rectangular sections

=  $\frac{A_s}{b_w d}$  for flanged sections

$A_s$  = area of steel on either face

- (ii) the reinforcement ratio  $p$  should not exceed 0.025.
- (2) At least two bars should be provided continuously both at top and bottom.
- (3) The positive moment resistance at the face of a joint should not be less than one-half of the negative moment resistance provided at that face of the joint.

Design Tables 21.2 and 21.3 similar to Tables 5.4 and 6.2 have been generated using the above recommendations and may be used for the design of beam sections for ductility.

**Table 21.2 Tension and compression steel  $p_t$  and  $p_c$   
for M30 concrete and Fe 500 grade steel ( $p_t/p_c \leq 2.0$ )**

$M/bd^2$	$d_1/d = 0.05$		$d_1/d = 0.10$		$d_1/d = 0.15$		$d_1/d = 0.20$	
	$p_t$	$p_c$	$p_t$	$p_c$	$p_t$	$p_c$	$p_t$	$p_c$
1.00	0.237	0.119	0.000	0.000	0.000	0.000	0.000	0.000
1.20	0.286	0.143	0.000	0.000	0.000	0.000	0.000	0.000
1.40	0.335	0.167	0.000	0.000	0.000	0.000	0.000	0.000
1.60	0.385	0.193	0.000	0.000	0.000	0.000	0.000	0.000
1.80	0.434	0.217	0.444	0.222	0.000	0.000	0.000	0.000
2.00	0.484	0.242	0.494	0.247	0.000	0.000	0.000	0.000
2.20	0.534	0.267	0.546	0.273	0.000	0.000	0.000	0.000
2.40	0.584	0.292	0.598	0.299	0.611	0.306	0.000	0.000
2.60	0.633	0.316	0.650	0.325	0.665	0.332	0.000	0.000
2.80	0.684	0.342	0.702	0.351	0.718	0.359	0.000	0.000
3.00	0.735	0.367	0.756	0.378	0.772	0.386	0.000	0.000
3.20	0.786	0.393	0.809	0.405	0.827	0.414	0.847	0.423
3.40	0.837	0.418	0.862	0.431	0.882	0.441	0.904	0.452
3.60	0.888	0.444	0.915	0.457	0.938	0.469	0.960	0.480
3.80	0.939	0.470	0.969	0.484	0.995	0.497	1.017	0.509
4.00	0.991	0.495	1.022	0.511	1.051	0.526	1.076	0.538
4.20	1.044	0.522	1.076	0.538	1.108	0.554	1.136	0.568
4.40	1.095	0.548	1.130	0.565	1.165	0.583	1.194	0.597
4.60	1.150	0.575	1.186	0.593	1.224	0.612	1.255	0.627
4.80	1.202	0.601	1.240	0.620	1.281	0.640	1.315	0.657
5.00	1.256	0.628	1.297	0.648	1.338	0.669	1.376	0.688
5.20	1.309	0.654	1.352	0.676	1.396	0.698	1.438	0.719
5.40	1.363	0.682	1.409	0.704	1.455	0.728	1.501	0.751
5.60	1.417	0.709	1.466	0.733	1.515	0.757	1.563	0.782
5.80	1.472	0.736	1.522	0.761	1.573	0.786	1.628	0.814
6.00	1.526	0.763	1.580	0.790	1.633	0.816	1.691	0.846
6.20	1.582	0.791	1.637	0.818	1.693	0.846	1.753	0.876
6.40	1.639	0.819	1.694	0.847	1.753	0.876	1.817	0.908
6.60	1.693	0.847	1.753	0.876	1.813	0.907	1.881	0.940
6.80	1.749	0.875	1.812	0.906	1.875	0.937	1.944	0.972
7.00	1.808	0.904	1.871	0.935	1.937	0.969	2.002	1.042
7.20	1.864	0.932	1.930	0.965	2.000	1.000	2.058	1.111
7.40	1.920	0.960	1.989	0.995	2.059	1.042	2.116	1.181
7.60	1.979	0.989	2.050	1.025	2.113	1.104	2.174	1.251
7.80	2.037	1.019	2.110	1.056	2.168	1.166	2.231	1.321
8.00	2.095	1.048	2.162	1.112	2.221	1.227	2.288	1.390

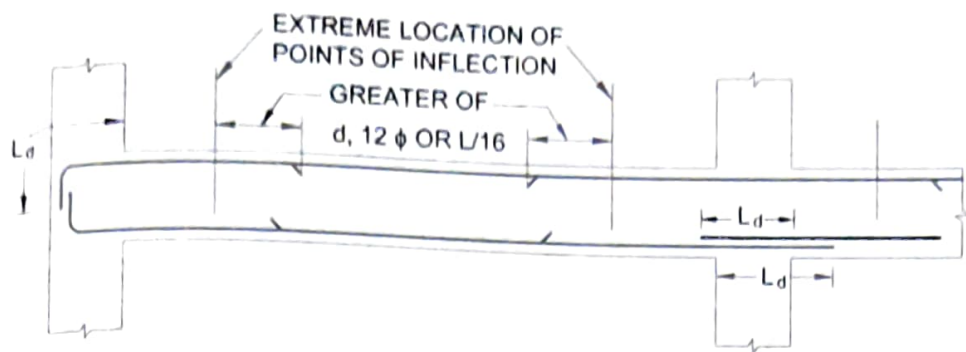


**Table 21.3 Tension and compression steel  $p_t$  and  $p_c$   
for M35 concrete and Fe 500 grade steel ( $p_t/p_c \leq 2.0$ )**

M/bd <sup>2</sup>	$d_1/d = 0.05$		$d_1/d = 0.10$		$d_1/d = 0.15$		$d_1/d = 0.20$	
	$p_t$	$p_c$	$p_t$	$p_c$	$p_t$	$p_c$	$p_t$	$p_c$
1.00	0.238	0.119	0.000	0.000	0.000	0.000	0.000	0.000
1.20	0.286	0.143	0.000	0.000	0.000	0.000	0.000	0.000
1.40	0.335	0.167	0.000	0.000	0.000	0.000	0.000	0.000
1.60	0.384	0.192	0.000	0.000	0.000	0.000	0.000	0.000
1.80	0.432	0.216	0.000	0.000	0.000	0.000	0.000	0.000
2.00	0.481	0.241	0.492	0.246	0.000	0.000	0.000	0.000
2.20	0.532	0.266	0.542	0.271	0.000	0.000	0.000	0.000
2.40	0.580	0.290	0.594	0.297	0.000	0.000	0.000	0.000
2.60	0.629	0.315	0.646	0.323	0.000	0.000	0.000	0.000
2.80	0.681	0.340	0.697	0.349	0.712	0.356	0.000	0.000
3.00	0.730	0.365	0.749	0.375	0.765	0.383	0.000	0.000
3.20	0.781	0.390	0.801	0.401	0.819	0.409	0.000	0.000
3.40	0.831	0.416	0.854	0.427	0.874	0.437	0.000	0.000
3.60	0.882	0.441	0.908	0.454	0.929	0.464	0.950	0.475
3.80	0.933	0.467	0.960	0.480	0.984	0.492	1.007	0.503
4.00	0.984	0.492	1.015	0.507	1.039	0.519	1.063	0.532
4.20	1.037	0.518	1.068	0.534	1.094	0.547	1.120	0.560
4.40	1.088	0.544	1.121	0.561	1.151	0.575	1.178	0.589
4.60	1.139	0.569	1.175	0.587	1.207	0.604	1.236	0.618
4.80	1.192	0.596	1.230	0.615	1.264	0.632	1.295	0.647
5.00	1.245	0.622	1.283	0.642	1.322	0.661	1.354	0.677
5.20	1.297	0.649	1.337	0.669	1.379	0.689	1.414	0.707
5.40	1.350	0.675	1.393	0.696	1.437	0.719	1.474	0.737
5.60	1.404	0.702	1.448	0.724	1.494	0.747	1.534	0.767
5.80	1.456	0.728	1.503	0.751	1.553	0.777	1.596	0.798
6.00	1.510	0.755	1.559	0.780	1.610	0.805	1.657	0.829
6.20	1.564	0.782	1.616	0.808	1.669	0.834	1.720	0.860
6.40	1.618	0.809	1.673	0.836	1.728	0.864	1.783	0.892
6.60	1.672	0.836	1.729	0.865	1.787	0.894	1.846	0.923
6.80	1.728	0.864	1.786	0.893	1.847	0.923	1.910	0.955
7.00	1.782	0.891	1.843	0.921	1.907	0.953	1.973	0.986
7.20	1.838	0.919	1.901	0.951	1.966	0.983	2.036	1.018
7.40	1.894	0.947	1.958	0.979	2.026	1.013	2.098	1.049
7.60	1.950	0.975	2.017	1.008	2.088	1.044	2.162	1.081
7.80	2.006	1.003	2.076	1.038	2.148	1.074	2.226	1.113
8.00	2.063	1.031	2.135	1.067	2.209	1.105	2.287	1.166

- (4) Neither the negative nor the positive moment resistance at any section along the member length should be less than one-fourth of the maximum moment resistance provided at the face of either joint.
- (5) The detailing rules as discussed in Chapter 9 are adequate and should be carefully followed.
- (6) When a beam frames into a column, both the top and bottom bars of the beam should be anchored into the column so as to develop their full strength in bond beyond the section of the beam at the face of the column. Where beams exist on

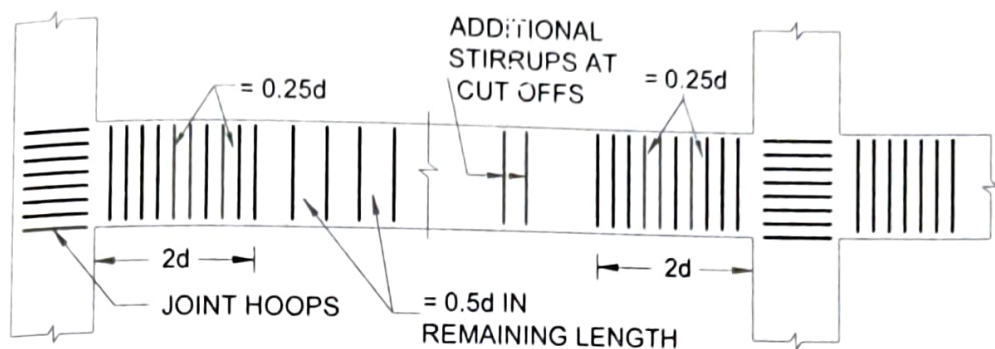
both sides of the column, both face bars of beams must be taken continuously through the column as shown in Fig. 21.9. To avoid congestion of steel in a column in which the beam frames on one side only, the use of hair pin type of bars spliced outside the column instead of anchoring the bars in the column is suggested.



**Fig. 21.9 Detailing of main reinforcing bars in beams**

### Shear Reinforcement

- (7) The design of shear stirrups follow normal practice as discussed in Chapter 8. The spacing of the vertical stirrups should not exceed  $0.25d$  in a length equal to  $2d$  near each end of the beam and  $0.5d$  in the remaining length of the beam as shown in Fig. 21.10.



**Fig. 21.10 Detailing of shear stirrups**

### (B) Columns

A column should satisfy the following requirements:

- (1) The factored axial stress  $P/A$  on a column under earthquake condition is more than  $0.1 \sigma_{ck}$ .
- (2) The ratio of shortest cross-sectional dimension to the perpendicular dimension is not less than 0.4.
- (3) The minimum dimension of the member is 200 mm. However, in frames where the beam span exceeds 5 m c/c, or the columns having unsupported length exceeds 4 m, the shortest dimension of a column should not be less than 300 mm.

### Special Confining Reinforcement

- (1) If  $P/A \geq 0.1 \sigma_{ck}$ , special confining reinforcement is required at the column ends :
  - (i) The cross-sectional area of bars forming circular hoops or spirals used for confinement of concrete is given by:



$$a_{sp} = 0.09 S D_c \left( \frac{A_g}{A_c} - 1 \right) \frac{\sigma_{ck}}{\sigma_{sp}} \quad (21.8)$$

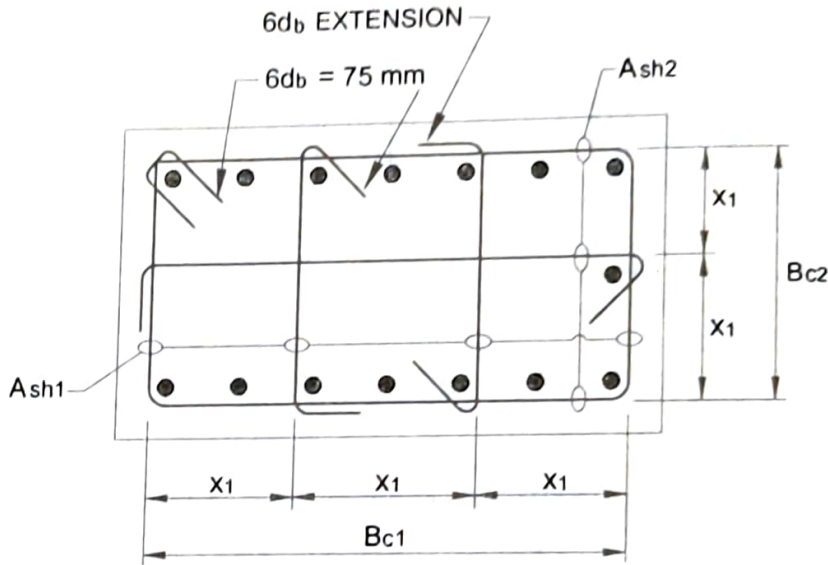
This is the same as Eq. 16.7d.

- (ii) Clause 7.4.7 of IS:13920 gives two equations to determine area of one leg of the special confinement reinforcement in a column or wall section. A cross-tie may be provided in order to reduce the value of spacing 'h'. In the case of rectangular closed stirrups used in rectangular sections, the area of ties is given by:

$$a_{sp} = 0.18 S h \left( \frac{A_g}{A_c} - 1 \right) \frac{\sigma_{ck}}{\sigma_{sp}} \quad (21.9)$$

where  $h$  = longer dimension of the rectangular confining stirrup

The maximum value of 'h' is 300 mm. However, a cross-tie must engage peripheral longitudinal bars. The above equation was adopted from the then ACI 318 code. This equation has been revised. Clause 21.6.4.4 of the ACI 318-2008 provides a set of two equations to compute total area of transverse reinforcement per unit height rather than the area of one leg only. These equations are independent of 'h' but its value is restricted to 350 mm. The detailing of ties is shown in Fig. 21.11. It should be noted that consecutive cross ties engaging the same longitudinal bar have their 90° hooks on opposite sides of the column.



**Fig. 21.11 Detailing of lateral ties in a column**

$$A_{sh} = 0.3 S b_c \frac{f'_c}{f_y} \left[ \frac{A_g}{A_{ch}} - 1.0 \right] \quad (21.10a)$$

$$A_{sh} = 0.09 S b_c \frac{f'_c}{f_y} \quad (21.10b)$$

where

$b_c$  = Cross sectional dimension of member core measured to the outside edges of transverse reinforcement

$h$  = Longer dimension of the rectangular confining hoop measured to its outer

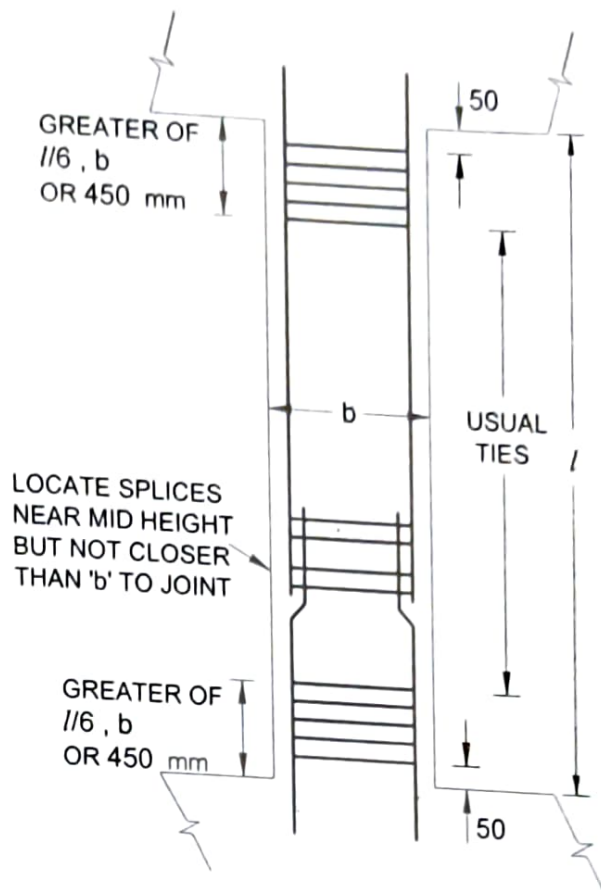
$f_{ck}, f_c$  = Characteristic compressive strength of concrete cube,

$f_y$  = Yield stress of steel used in hoops

$S$  = Spacing of hoops

Equations 21.10a and 21.10b are to be satisfied in both cross sectional directions of the rectangular core. For each direction,  $b_c$  is the core dimension perpendicular to the tie legs that constitute  $A_{sh}$ .

- (2) The special confining steel where required must be provided above and below the beam connections as shown in Fig. 21.12, in a length of the column at each end which is largest of the following :



**Fig. 21.12 Spacing of lateral ties**

- (i)  $1/6$  of the clear height of the column,
- (ii) larger lateral dimension of the column, and
- (iii) 450 mm.

The pitch of lateral ties should not exceed  $1/4$ th of the minimum member dimension nor 100 mm.

- (3) When the calculated point of contra-flexure under the effect of gravity and earthquake loads is not within the middle half of the member clear height, special confinement reinforcement must be provided over the full height of the column.

Clause 5.4.3.2 of Eurocode 1998-1 requires that if  $l_c/h_c < 3$ , the entire height of the primary seismic column shall be considered as being a critical region and shall be reinforced accordingly,

where

$h_c$  = the largest cross-sectional dimension of the column (in metres)

$l_{cl}$  = the clear length of the column (in metres).

- (4) Shear reinforcement must be provided in columns to resist the nominal shear resulting from the lateral and vertical loads at limit state of collapse of the frame. Shear strength of columns increases in the presence of compressive axial loads. Clause 40.2.2 of IS: 456 requires that for members subjected to axial compression  $P_u$  the shear strength of concrete  $\tau'_c$  is given by :

$$\tau'_c = \delta \tau'_c \quad (21.11a)$$

$$\text{where,} \quad \delta = 1 + \frac{3P_u}{A_g \sigma_{ck}} \quad (21.11b)$$

$P_u$  = axial load in N

$A_g$  = gross area of the concrete section in  $\text{mm}^2$

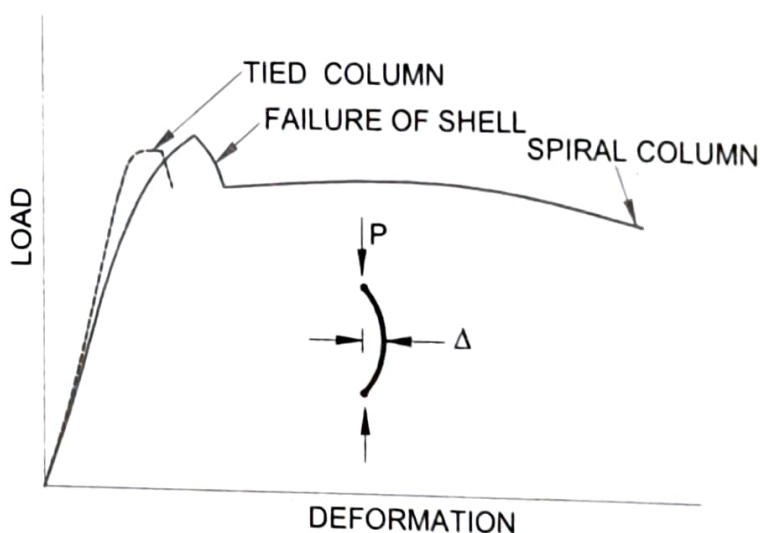
$\tau_c$  = shear strength of concrete as given in Table 8.2

The spacing of shear reinforcement should not exceed  $0.5d$ , where  $d$  is the effective depth of column measured from compression fibre to the tension steel.

- (5) Spiral columns should be used where ever possible especially if  $P_u > 0.4 P_b$

where,  $P_b$  = balance axial load

Figure 21.13 shows that spiral columns are much more ductile as compared with columns with lateral ties.



**Fig. 21.13 Behaviour of a tied and a spiral column**

### (C) Beam-Column Connections

The beam-column joints are generally the weakest links in a structure. To avoid frame failure due to inadequate joints, the joint details must be carefully considered as discussed in section 19.15. The following points need special attention :

- (1) Anchorage of beam reinforcement in the joint – The part of beam longitudinal reinforcement bent in joints for anchorage shall always be placed inside the corresponding column hoops.





- (1) The ratio of larger cross-section dimension to the shorter dimension shall be greater than 4.
- (2) The thickness of any part of the wall shall preferably, not be less than 150 mm.
- (3) The effective flange width, to be used in the design of flanged wall sections, shall be assumed to extend beyond the face of the web for a distance which shall be the smaller of
  - (a) half the distance to an adjacent shear wall web, and
  - (b) 1/10th of the total wall height.
- (4) The horizontal reinforcement in walls should not be less than that required from shear strength criterion.

For shear walls, the ACI code requires that the confinement steel should be determined using Eq 21.10b and not Eq. 21.10a that provides much less confining steel in walls than that obtained from Eq. 21.10a.

#### *Boundary Elements*

- (1) Boundary elements are portions along the wall edges that are strengthened by longitudinal and transverse reinforcement. Though they may have the same thickness as that of the wall web it is advantageous to provide them with greater thickness. IS : 13920 is currently silent on how to determine length and width of a boundary element. In the absence of this information, it has to be decided based on experience and judgement.
- (2) Where the extreme fibre compressive stress in the wall due to factored gravity loads plus factored earthquake force exceeds  $0.2\sigma_{ck}$ , boundary elements should be provided along the vertical boundaries of walls. The boundary elements may be discontinued where the calculated compressive stress becomes less than  $0.15 \sigma_{ck}$ . The compressive stress should be calculated using a linearly elastic model and gross section properties.
- (3) A boundary element should have adequate axial load carrying capacity, assuming short column action, so as to enable it to carry an axial compression equal to the sum of factored gravity load on it and the additional compressive load induced by the seismic force.

The latter may be calculated as 
$$\frac{M_u - M_{uv}}{C_w} \quad (21.12)$$

where

$M_u$  = factored design moment on the entire wall section,

$M_{uv}$  = moment of resistance provided by distributed vertical reinforcement across the wall section, and

$C_w$  = center to center distance between the boundary elements along the two vertical edges of the wall.

- (4) If the gravity load adds to the strength of the wall, its load factor should be taken as 0.8.
- (5) The percentage of vertical reinforcement in the boundary elements should not be less than 0.8 percent, nor greater than 6 percent. In order to avoid congestion, the practical upper limit would be 4 percent.



- (6) Boundary elements, where required, should be provided throughout their height with special confining reinforcement as per Eq. 21.9.
- (7) Boundary elements need not be provided, if the entire wall section is provided with special confining reinforcement as per Eq. 21.9.

For a boundary element, clause 21.9.6.2 of ACI 318 specifies how to determine the depth of neutral axis  $N_d$ :

$$N_d > \frac{l}{600(\delta_u/h)} \quad (21.13)$$

where,

$\delta_u$  = design displacement,  $l$  = length of shear wall

For a given factored axial force, moment capacity is determined using appropriate P-M interaction curve for a column, that is, boundary element. Thus depth  $N_d$  is calculated for the factored axial force and moment capacity consistent with the design displacement  $\delta_u$ . The ratio  $\delta_u/h$  should not be taken less than 0.007. This helps determine length of boundary element.

The thickness of the boundary element can be determined as per clause 5.4.3.4 of EC 8 - Part 1 as follows:

$$\text{If } l_b > 2 t_b \text{ and } 0.2 l_w, t_b > h/10$$

$$\text{If } l_b < 2 t_b \text{ and } 0.2 l_w, t_b > h/15$$

where

$h$  = storey height or height of shear wall

$l_b$  = length of boundary element

$t_b$  = thickness of boundary element.

The design of foundation of a shear wall needs more attention as it may be under uplift due to large bending moment at its base.

## 21.7 ILLUSTRATIVE EXAMPLES

### Example 21.1

A R.C. frame consists of beams having spans of 6 m c/c. A typical floor inner beam carries a negative bending moment of 450 kNm and a shear of 325 kN at the face of beam column joint due to gravity and earthquake loads. Design the beam section for ductility.

*Solution*

Let the cross section of beam = 350 × 650 mm

Effective cover for tension steel = 75 mm

Factored bending moment = 450 × 1.2 = 540 kNm

$$\therefore \frac{M_u}{bd^2} = \frac{540 \times 10^6}{350 \times 575^2} = 4.67$$

The section can be designed as a doubly reinforced section. For M20 concrete and Fe 415 D grade steel, ( $d'/d = 0.075$ ) or  $d' \approx 40$  mm

where  $d'$  = effective cover for compression steel

Tension steel  $p_t$  = 1.53 % and

$$\text{Compression steel } p_c = 0.60 \%$$

$$\begin{aligned} \text{Minimum tension steel } p_{\min} &= \frac{0.24 \sqrt{\sigma_{ck}}}{\sigma_y} = \frac{0.24 \sqrt{20}}{415} \\ &= 0.00259 \text{ or, } 0.26 \% \end{aligned}$$

OK

$$\text{Maximum tension steel } p_{\max} = 2.5 \% > 1.53 \%$$

OK

However, the positive moment capacity is less than 50% of the negative moment capacity at the same face. Thus, it violates the ductility provisions of IS : 13920-1993. Let us redesign the section as a doubly reinforced section such that the compression steel is at least 50 % of the tension steel at this section.

$$d'/d = 0.075,$$

$$p_t = 1.48 \% = 29.87 \text{ cm}^2$$

$$\text{and } p_c = 0.74 \% = 14.89 \text{ cm}^2$$

Provide 5-28  $\phi$  bars at the top face ( $A_t = 30.78 \text{ cm}^2$ ) and 3 - 28  $\phi$  bars at the bottom face ( $A_{sc} = 18.47 \text{ cm}^2$ ).

*Shear force*

$$\text{Factored shear force } V_u = 1.2 \times 325 = 390 \text{ kN}$$

Shear strength of concrete for 1.48 % tension steel,

$$\tau_c = 0.72 \text{ MPa}$$

$$\text{Nominal shear stress } \tau_v = \frac{V_u}{bd} = \frac{390 \times 1000}{350 \times 575}$$

$$= 1.94 \text{ MPa} < 2.8 \text{ MPa}$$

OK

Let us choose 8  $\phi$  - 4 legged stirrups. The spacing of stirrups is given by

$$x = \frac{0.8 / \sigma_y A_{sv}}{(\tau_v - \tau_c) b}$$

$$x = \frac{0.87 \times 415 \times 4 \times 50}{(1.94 - 0.72) \times 350} = 169 \text{ mm}$$

$$\star \frac{d}{4} = \frac{575}{4} = 143.75 \text{ mm}$$

$$\star 8 \times 28 \phi = 224 \text{ mm}$$

$$\star 100 \text{ mm}$$

Let us provide 8 mm - 4 legged stirrups @ 125 mm c/c in a length equal to  $2d \approx 1150 \text{ mm}$  from the face of the beam column joint. The reinforcement detail is shown in Fig. 21.15. The first stirrup can be provided at a distance of 25 mm from the face of the joint so that the remaining spacing is 1125 mm. It will require a total of 10 stirrups.

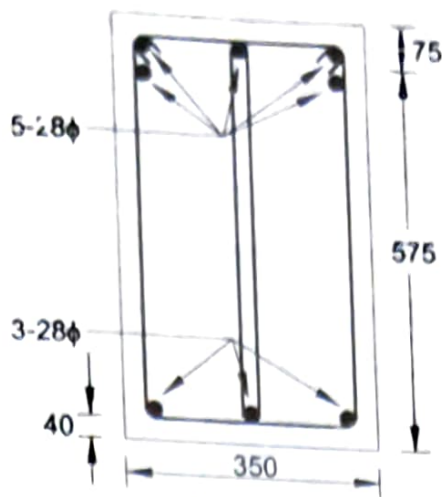


Fig. 21.15

### Example 21.2

A column in a multistory R. C. building is subjected to an axial force of 2500 kN and a bending moment of 650 kNm under gravity and earthquake loads. Design the column section for ductility. Use M20 concrete and Fe 415 D grade steel.

*Solution*

Let the column cross-section = 600 mm × 600 mm

Factored bending moment  $M_u = 1.2 \times 650 = 780 \text{ kNm}$

Factored axial force  $P_u = 1.2 \times 2500 = 3000 \text{ kN}$

$$\therefore \frac{P_u}{\sigma_{ck} bD} = \frac{3000 \times 10^3}{20 \times 600^2} \approx 0.417$$

$$\frac{M_u}{\sigma_{ck} bD^2} = \frac{780 \times 10^6}{20 \times 600^3} = 0.18$$

Let us use chart 44 of SP : 16 Handbook for columns reinforced on all the four faces,

For,  $d'/D = 0.10$

$$\frac{P}{\sigma_{ck}} = 0.162 \text{ or } p = 3.24 \% < 4 \% \quad \text{OK}$$

Area of steel required = 116.64 cm<sup>2</sup>

Provide 16-32  $\phi$  bars (= 128.68 cm<sup>2</sup>).

*Lateral steel*

$$\text{Diameter of ties} = \frac{\phi_L}{4} = \frac{32}{4} = 8 \text{ mm}$$

$$\begin{aligned} \text{Maximum pitch} &< \frac{D}{4} = \frac{600}{4} = 150 \text{ mm} \\ &< 100 \text{ mm} \end{aligned}$$

Confining reinforcement is given by Eq. 21.9

$$a_{sp} = 0.18 p h \left( \frac{A_g}{A_c} - 1 \right) \frac{\sigma_{ck}}{\sigma_y}$$



Clear cover to stirrups = 40 mm

$h$  = longer dimension of the rectangular confining hoop measured to its outer face

$$= 600 - 40 - 40 = 520 \text{ mm}$$

$$> 300 \text{ mm}$$

NG

Therefore, provide a cross tie around the middle longitudinal bar (Fig. 21.16a)

Thus, revised  $h = 520/2 = 260 \text{ mm}$

$$< 300 \text{ mm}$$

OK

Area of concrete core  $A_c = (600 - 40 - 40)^2 = 520 \times 520 \text{ mm}^2$

Gross area of concrete  $A_g = 600 \times 600 \text{ mm}^2$

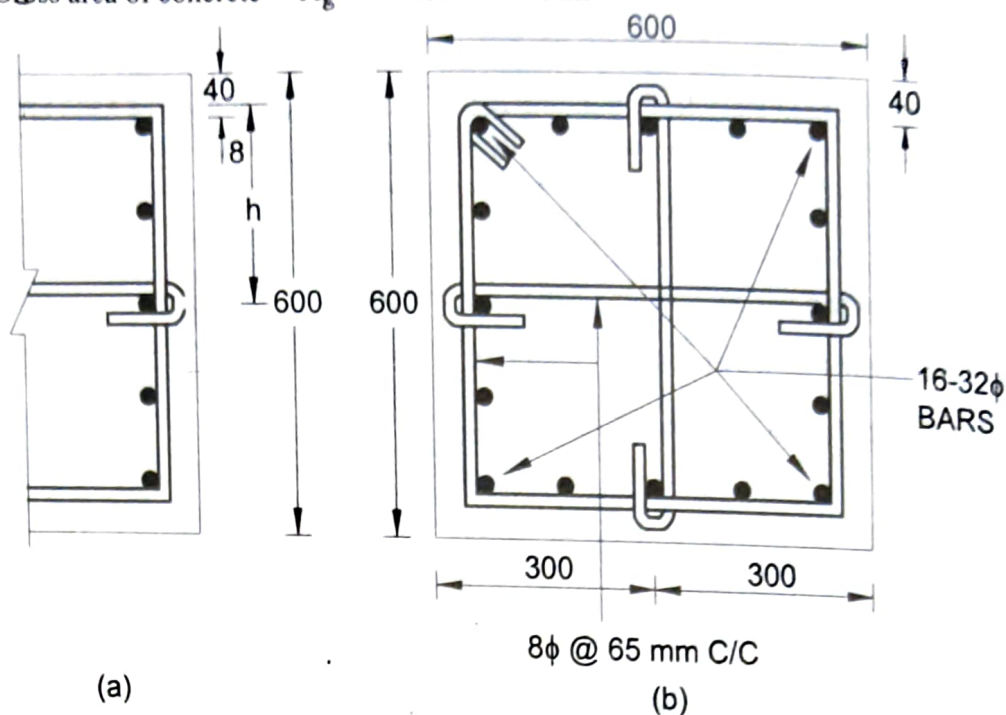


Fig. 21.16

$$\therefore \frac{A_g}{A_c} = 1.33$$

Area of one stirrup,  $a_{sp} = \frac{\pi}{4} 8^2 = 50 \text{ mm}^2$

Spacing of stirrup,  $p = \frac{a_{sp} \sigma_y / \sigma_{ck}}{0.18h \left[ \frac{A_g}{A_c} - 1 \right]} = \frac{50 \times 415 / 20}{0.18 \times 260 \times 0.33}$

or,  $p = 67 \text{ mm}$

Clear height of column = 3600 mm

Distance of special confinement is given by

$$y > \frac{L}{6} = \frac{3600}{6} = 600 \text{ mm}$$

$$> D = 600 \text{ mm}$$

$$> 450 \text{ mm}$$

$$\therefore \text{Adopt } y = 650 \text{ mm}$$

Let us provide 8 mm stirrups @ 65 mm c/c in a distance equal to 650 mm from the face of the joint.

Elsewhere, the pitch is given by  $p < 600 \text{ mm}$

$$< 48 \times 8 = 384 \text{ mm}$$

$$< 300 \text{ mm.}$$

That is, adopt 300 mm c/c.

The reinforcement detail is shown in Fig. 21.16b.

### Example 21.3

Consider an inner beam-column joint in the ground floor roof of a eight storey building in Noida, U.P. The data are as follows:

Grade of concrete = M25, steel grade = Fe 415 D

Clear span of beam to the left side of the joint = 4.5 m

Clear span of beam to the right side of the joint = 4 m

Slab thickness = 125 mm, finish on slab = 50 mm thick

Live load on floor =  $2 \text{ kN/m}^2$ ;

Wall thickness on beams = 115 mm

The axial load in column at the joint = 900 kN

Beam size = 230 mm  $\times$  550 mm with 1.5% steel at top (3 – 25 mm and 2 – 16 mm) and 0.8% steel at bottom on either side of the joint.

Column size = 230 mm  $\times$  650 mm with 3.46% steel (8 – 25mm and 4 – 20 mm bars)

Check if the joint satisfies weak girder-strong column proportion. Also check the shear in beam and column.

*Solution*

(a) Let us first examine the beam-column joint in bending

*Bending of column about weak axis*

From Table 6.2 for doubly reinforced sections, for a given amount of reinforcement,

Hogging moment capacity of beam  $\frac{M_u}{bd^2} = 4.45$

$$\text{or, } M_u = 4.45 \times 230 \times 500 \times 500 = 255.87 \text{ kNm}$$

From Table 5.4 for singly reinforced sections, for a given amount of reinforcement,

Sagging moment capacity of the beam  $\frac{M_u}{bd^2} = 2.50$

$$\text{or, } M_u = 2.50 \times 230 \times 500 \times 500 = 143.75 \text{ kNm}$$

From Fig. 16.15

$$\frac{P_u}{\sigma_{ck} b D} = \frac{1.2 \times 900 \times 1000}{25 \times 650 \times 230} = 0.29$$

$$p/\sigma_{ck} = 3.46/25 = 0.138$$

$$\therefore \frac{M_u}{\sigma_{ck} b D^2} = 0.225 \quad \text{or,} \quad M_u = 193.4 \text{ kNm}$$

$$\Sigma M_g = 255.87 + 143.75 = 399.60 \text{ kNm}$$

$$\Sigma M_c = 2 \times 193.4 = 386.8 \text{ kNm}$$

$$\Sigma M_c < 1.2 \Sigma M_g$$

hence, the beam-column joint is not based on weak girder-strong column proportions. There is a need to increase width of column.

(b) Let us now examine the shear capacity of beam on left side of the joint

$$\text{Dead load intensity on beam} = 28.5 \text{ kN/m (given)}$$

$$\text{Live load intensity on beam} = 7 \text{ kN/m}$$

$$\text{Factored shear due to gravity} = 1.2 (28.5 + 7) \times 4.5/2 = 95.85 \text{ kN}$$

$$\text{Shear due to formation of plastic hinge in beam} = 1.4 \times 399.6/4.5 = 124.32 \text{ kN}$$

$$\text{Total } V_u = 220 \text{ kN}$$

$$\text{Nominal shear stress } \tau_v = 220 \times 1000/230 \times 500 = 1.91 \text{ MPa}$$

$$\text{Shear strength of concrete } \tau_c = 0.72 \text{ MPa for 1.5\% tension steel}$$

Provide 10 mm – 2 legged stirrups @ 150 mm c/c

OK

(c) Let us now examine the shear capacity of column

$$\text{Storey height} = 3.25 \text{ m}$$

$$\text{Factored shear in column} = 1.4 \times 399.6/3.25 = 172 \text{ kN}$$

$$\text{Nominal shear stress } \tau_v = 172 \times 1000/230 \times 650 = 1.15 \text{ MPa}$$

$$\text{Shear strength of concrete } \tau_c = 0.77 \text{ MPa for 1.7\% tension steel on one face}$$

Increase in shear strength as per IS: 456 – 2000,

$$\delta = 1 + \frac{3P_u}{A_g \sigma_{ck}} = 1 + \frac{3 \times 1.2 \times 900 \times 1000}{230 \times 650 \times 25} = 1.87$$

$$\text{Increased shear strength of concrete} = 1.87 \times 0.77 = 1.43 \text{ MPa} > 1.15 \text{ MPa}$$

OK

### Example 21.4

Consider an inner beam column joint in the ground floor roof of a six storey building to be built in Delhi. The data are as follows:

$$\text{Grade of concrete} = \text{M20, steel grade} = \text{Fe 415 D}$$

$$\text{Clear span of beam to either side of the joint} = 3.5 \text{ m}$$

$$\text{Slab thickness} = 125 \text{ mm, Finish on slab} = 50 \text{ mm thick}$$

$$\text{Live load on floor} = 2 \text{ kN/m}^2; \text{ Wall thickness on beams} = 230 \text{ mm}$$

$$\text{The axial load in column at the joint} = 1000 \text{ kN}$$

$$\text{Beam size} = 230 \text{ mm} \times 600 \text{ mm with 0.75\% steel at top (3-20 mm) and 0.47\% steel at bottom on either side of the joint.}$$

Column size = 230 mm × 600 mm with 2.33% steel (4 – 25 mm and 4 – 20 mm bars)

Check if the joint satisfies weak girder-strong column proportion. Also, check the shear capacity of beam.

*Solution*

(a) Let us first examine the beam-column joint in bending

*Bending of column about weak axis*

From Table 5.4 for singly reinforced sections, for a given amount of reinforcement,

$$\text{Hogging moment capacity of beam } \frac{M_u}{bD^2} = 2.28$$

$$\text{or, } M_u = 2.28 \times 230 \times 600 \times 600 = 158.60 \text{ kNm}$$

From Table 5.4 for singly reinforced sections, for a given amount of reinforcement,

$$\text{Sagging moment capacity of beam } \frac{M_u}{bD^2} = 1.54$$

$$\text{or, } M_u = 1.54 \times 230 \times 600 \times 600 = 107.1 \text{ kNm}$$

From Fig. 16.15 for columns

$$\frac{P_u}{\sigma_{ck} bD} = \frac{1.2 \times 1000 \times 1000}{20 \times 600 \times 230} = 0.434$$

$$p/\sigma_{ck} = 2.33/20 = 0.117$$

$$\frac{M_u}{\sigma_{ck} bD^2} = 0.155 \text{ or, } M_u = 98.4 \text{ kNm}$$

$$\Sigma M_g = 158.6 + 107.1 = 266 \text{ kNm}$$

$$\Sigma M_c = 2 \times 98.4 = 196.8 \text{ kNm}$$

$$\Sigma M_c < 1.2 \Sigma M_g$$

Hence, the beam-column joint is not based on weak girder-strong column proportions. There is a need to increase width of column.

(b) Let us now examine the shear capacity of beam

$$\text{Dead load intensity on beam} = 30 \text{ kN/m}$$

$$\text{Live load intensity on beam} = 7 \text{ kN/m}$$

$$\text{Factored shear due to gravity} = 1.2 (30 + 7) \times 3.5/2 = 77.7 \text{ kN}$$

$$\text{Shear due to formation of plastic hinge in beam} = 1.4 \times 266/3.5 = 106.4 \text{ kN}$$

$$\text{Total } V_u = 184 \text{ kN}$$

$$\text{Nominal shear stress } \tau_v = 184 \times 1000/230 \times 550 = 1.45 \text{ MPa}$$

$$\text{Shear strength of concrete } \tau_c = 0.56 \text{ MPa for } 0.75\% \text{ tension steel}$$

Provide 8 mm – 2 legged stirrups @ 150 mm c/c

OK



**Example 21.5**

Design a ductile flexural wall for a 20 storey building whose data is given in Table 21.4 and Fig. 21.17. Use M35 concrete and Fe 415 D grade steel.

**Table 21.4 Design forces for the flexural wall**

Story	Location	Axial load, kN			Shear force, V2,	Moment M3
		Service load			Factored	Factored
		Self wt	Live	Total	kN	kN-m
STORY20	Top	301.82	238.71	540.53	- 247.16	- 398.171
	Bottom	407.84	238.71	646.55	- 247.16	- 1139.64
STORY19	Top	706.76	474.8	1181.56	- 12.45	- 1563.31
	Bottom	812.78	474.8	1287.58	- 12.45	- 1600.65
STORY18	Top	1112.39	711.6	1823.99	52.13	- 2039.6
	Bottom	1218.42	711.6	1930.02	52.13	- 1883.23
STORY17	Top	1518.06	948.53	2466.59	133.24	- 2345.48
	Bottom	1624.09	948.53	2572.62	133.24	- 1945.75
STORY16	Top	1923.93	1185.77	3109.7	198.03	- 2432.85
	Bottom	2029.96	1185.77	3215.73	198.03	- 1838.76
STORY15	Top	2330.04	1423.41	3753.45	254.75	- 2351.03
	Bottom	2436.07	1423.41	3859.48	254.75	- 1586.79
STORY14	Top	2736.43	1661.5	4397.93	303.6	- 2122.73
	Bottom	2842.45	1661.5	4503.95	303.6	- 1211.94
STORY13	Top	3143.14	1900.13	5043.27	346.18	- 1768.64
	Bottom	3249.17	1900.13	5149.3	346.18	- 730.092
STORY12	Top	3550.22	2139.37	5689.59	383.72	- 1303.41
	Bottom	3656.25	2139.37	5795.62	383.72	- 152.266
STORY11	Top	3957.72	2379.31	6337.03	417.41	- 736.969
	Bottom	4063.75	2379.31	6443.06	417.41	515.261
STORY10	Top	4365.68	2621.02	6985.7	448.43	- 74.567
	Bottom	4471.71	2621.02	7091.73	448.43	1270.722
STORY9	Top	4774.15	2861.59	7635.74	477.94	683.006
	Bottom	4880.17	2861.59	7741.76	477.94	2116.821
STORY8	Top	5183.16	3104.09	8287.25	507.12	1539.453
	Bottom	5289.19	3104.09	8393.28	507.12	3060.816
STORY7	Top	5592.76	3347.61	8940.37	537.21	2503.095
	Bottom	5698.79	3347.61	9046.4	537.21	4114.723
STORY6	Top	6003	3592.22	9595.22	569.53	3587.129
	Bottom	6109.03	3592.22	9701.25	569.53	5295.715
STORY5	Top	6413.92	3838.02	10251.94	605.46	4810.079
	Bottom	6519.95	3838.02	10357.97	605.46	6626.448
STORY4	Top	6825.56	4085.07	10910.63	646.78	6196.189
	Bottom	6931.58	4085.07	11016.65	646.78	8136.516
STORY3	Top	7237.97	4333.5	11571.47	694.31	7776.934
	Bottom	7344	4333.5	11677.5	694.31	9859.872
STORY2	Top	7651.07	4583.27	12234.34	754.14	9588.516
	Bottom	7757.1	4583.27	12340.37	754.14	11850.93
STORY1	Top	8065.58	4835.03	12900.61	810.1	11688.19
	Bottom	8189.28	4835.03	13024.31	810.1	14523.55

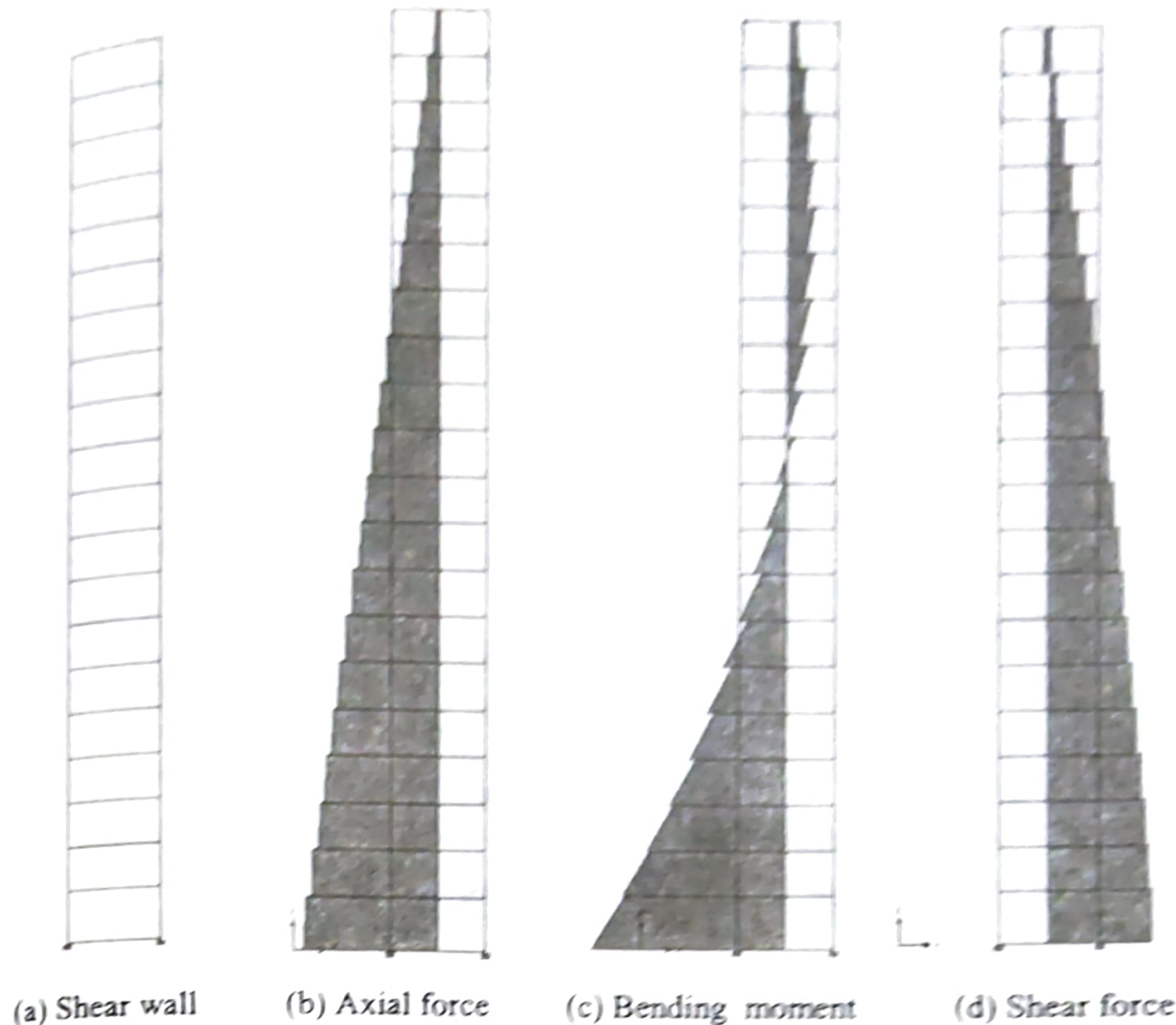


Fig. 21.17

*Solution*

Let us adopt a  $6000 \times 200$  mm cross section for the shear wall as shown in Fig. 21.18a.

(a) *Boundary element requirement check*

Stress in extreme fibre due to combined action of axial load and moments in  $6000 \text{ mm} \times 200 \text{ mm}$  rectangular section is

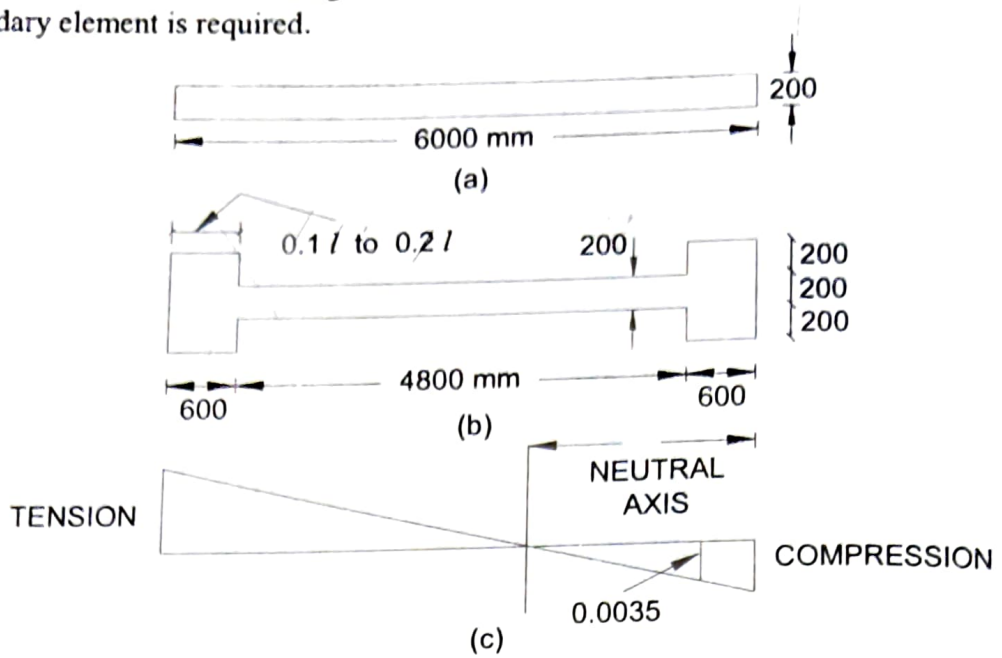
$$\sigma = \frac{P_u}{A_g} + \frac{M_u (l/2)}{I}$$

From table, design force at the base of wall,

$$\begin{aligned} P_u &= 1.2 \times 13024 = 15628 \text{ kN}, M_u = 14523 \text{ kNm}, l = 6000 \text{ mm} \\ &= \frac{15628 \times 10^3}{6000 \times 200} + \frac{14523 \times 10^6 \times (6000/2)}{3.6 \times 10^{12}} \\ &= 0.72 \sigma_{ck} > 0.2 \sigma_{ck} \end{aligned}$$

If stress in extreme fibre exceeds the limit, boundary element should be provided. Considering the advantage of providing thicker section at the ends, thickness is increased from 200 mm to 600 mm for a length of 600 mm at both the ends. Thus a boundary

element of  $600 \times 600$  mm size is provided at both ends as shown in Fig. 21.18b. The stress ratio calculated according to thickened section will decide the height upto which the boundary element is required.



**Fig. 21.18**

(b) Check for section requirements

Thickness of web provided = 200 mm > 150 mm

OK  
(clause 9.1.2)

(c) Check for reinforcement requirements

Minimum area of concentrated reinforcement at web in both directions

$$\begin{aligned} A_{st} &= 0.0025 t_w l_w, \quad l_w = \text{length of web} \\ &= 0.0025 \times 200 \times 4800 = 2400 \text{ mm}^2 \end{aligned}$$

Thickness of web,  $t_w = 200$  mm so reinforcement should be provided in two layers  
(clause 9.1.5)

Provide 8 mm bar @ 200 mm c/c in two layers in both horizontal and vertical directions.

The maximum spacing of reinforcement, (clause 9.1.7)

$$< l_w/5 = 4800/5 = 960 \text{ mm}$$

$$< 3 t_w = 3 \times 200 = 600 \text{ mm} < 450 \text{ mm} \quad \text{OK}$$

Maximum area of concentrated reinforcement at boundary element is

(clause 9.4.4)

$$A_{st} < 0.06 \text{ of gross area of boundary element}$$

$$= 0.06 \times 600 \times 600 = 21600 \text{ mm}^2$$

$$\text{Minimum area of reinforcement} = 0.008 \times 600 \times 600 = 2880 \text{ mm}^2$$

(clause 9.4.4)

$$\text{Provided minimum } 16 - 16 \text{ mm bars} = 3216 \text{ mm}^2 > 2880 \text{ mm}^2$$

OK

Maximum bar diameter

(clause 9.1.6)



In 200 mm thick web,  $d_b = 200/10 = 20 \text{ mm} > 8 \text{ mm}$

OK

In 600 mm thick boundary element,  $d_b = 600/10 = 60 \text{ mm} > 16 \text{ mm}$

OK

The area of each boundary element is 21.4 % of the total area and area of web alone is 57.2 %.

(d) Design for Shear

Design shear force at the base of wall,  $V_u = 810 \text{ kN}$

The nominal shear stress

$$\tau_v = \frac{V_u}{t_w l_w} = \frac{810 \times 10^3}{200 \times 4800} = 0.844 \text{ MPa}$$

The shear capacity of section for 0.25% reinforcement for M35 concrete =  $0.37 \text{ N/mm}^2$

So shear reinforcement need to be provided to carry a shear force

$$\begin{aligned} &= V_u - \tau_c b d = 810 \times 10^3 - 0.37 \times 4800 \times 200 \\ &= 455 \text{ kN} \end{aligned}$$

Horizontal reinforcement per m height

$$= \frac{455 \times 10^3 \times 1000}{0.87 \times 415 \times 4800} = 262 \text{ mm}^2/\text{m}$$

So provide minimum 0.25% reinforcement =  $50 \text{ mm}^2/\text{m}$

OK

Flexural strength of web

$$P_u = \text{Axial compression on wall} = 1/2 \times 13024 = 15628 \text{ kN}$$

$$\text{Axial load on web} = 0.572 \times 15628 = 8939 \text{ kN}$$

$$\frac{0.87 \sigma_y}{0.0035 E_s} = \frac{0.87 \times 415}{0.0035 \times 2 \times 10^5} = 0.515$$

$$\text{Vertical reinforcement ratio } \rho = \frac{A_{st}}{t_w l_w} = \frac{2500}{200 \times 4800} = 0.00261$$

$$\phi = \frac{0.87 \sigma_y \rho}{\sigma_{ck}} = \frac{0.87 \times 415 \times 0.00261}{35} = 0.027$$

$$\lambda = \frac{P_u}{\sigma_{ck} t_w l_w} = \frac{8939 \times 10^3}{35 \times 200 \times 4800} = 0.266$$

$$\frac{x}{l_w} = \left( \frac{\phi + \lambda}{2\phi + 0.36} \right) = \frac{0.027 + 0.266}{2 \times 0.027 + 0.36} = 0.7077$$

$$\begin{aligned} \frac{x_m}{l_w} &= \left( \frac{0.0035}{0.0035 + 0.87 \sigma_y / E_s} \right) \\ &= \left( \frac{0.0035}{0.0035 + 0.87 \times 415 / (2 \times 10^5)} \right) = 0.66 \end{aligned}$$



$$\frac{x}{l_w} > \frac{x_m}{l_w} \text{ so this is flexural compression failure,}$$

Moment capacity of web is given by

$$\frac{M_{uv}}{\sigma_{ck} t_w l_w^2} = \left[ \alpha_4 \left( \frac{x}{l_w} \right) - \alpha_5 \left( \frac{x}{l_w} \right)^2 + \alpha_6 \right] \quad (16.27)$$

where

$$\begin{aligned} \alpha_4 &= \left[ 0.18 + \frac{\phi}{2} \left( 1 - \frac{\beta}{2} - \frac{1}{2\beta} \right) \right] \\ &= \left[ 0.18 + \frac{0.027}{2} \left( 1 - \frac{0.515}{2} - \frac{1}{2 \times 0.515} \right) \right] \\ &= 0.176 \\ \alpha_5 &= \left[ 0.15 + \frac{\phi}{2} \left( 1 - \beta + \frac{\beta^2}{3} - \frac{1}{3\beta} \right) \right] \\ &= \left[ 0.15 + \frac{0.027}{2} \left( 1 - 0.515 + \frac{0.515^2}{3} - \frac{1}{3 \times 0.515} \right) \right] \\ &= 0.149 \\ \alpha_6 &= \frac{\phi}{12\beta \left( \frac{x}{l_w} \right)} \end{aligned}$$

where  $\frac{x}{l_w}$  calculated from Eq. 16.26

$$\alpha_1 \left( \frac{x}{l_w} \right)^2 + \alpha_2 \left( \frac{x}{l_w} \right) - \alpha_3 = 0 \quad (16.26)$$

where,

$$\begin{aligned} \alpha_1 &= \left[ 0.36 + \phi \left( 1 - \frac{\beta}{2} - \frac{1}{2\beta} \right) \right] \\ &= \left[ 0.36 + 0.027 \left( 1 - \frac{0.515}{2} - 0.266 \right) \right] = 0.3538 \\ \alpha_2 &= \left( \frac{\phi}{\beta} - \lambda \right) = \left( \frac{0.027}{0.515} - 0.266 \right) = -0.2135 \\ \alpha_3 &= \left( \frac{\phi}{2\beta} \right) = \left( \frac{0.027}{2 \times 0.515} \right) = 0.0262 \end{aligned}$$

$$0.3538 \left( \frac{x}{l_w} \right)^2 - 0.2135 \left( \frac{x}{l_w} \right) - 0.0262 = 0$$

$$\text{or} \quad \frac{x}{l_w} = 0.7080, -0.1045$$

Two values of  $x/l_w$  give two values of  $\alpha_6$  and  $M_{uv}$ .

$$\alpha_6 = \frac{\phi}{12\beta \left( \frac{x_u}{l_w} \right)} = \frac{0.027}{12 \times 0.515 \times (0.7080)} = 0.00617$$

and,

$$\alpha_6 = \frac{0.027}{12 \times 0.51 \times (-0.1045)} = -0.0416$$

$\therefore$  Eq. 16.27 gives,

$$M_{uv} = 35 \times 200 \times 4800^2 [0.17(0.7080) - 0.149(0.7080)^2 + 0.00617] = 9046 \text{ kNm}$$

and,

$$\begin{aligned} M_{uv} &= 35 \times 250 \times 4800^2 [0.176(-0.1045) - 0.149(-0.1045)^2 - 0.0416] \\ &= -12422 \text{ kNm} \end{aligned}$$

Therefore, adopt lower of the two values,  $M_{uv} = 9046 \text{ kNm}$

$$M_u - M_{uv} = 14523 - 9046 = 5477 \text{ kNm}$$

Center to center to boundary element  $C_w'' = 6 - 0.3 - 0.3 = 5.4 \text{ m}$

$\therefore$  Additional compressive force induced by the seismic force

$$\frac{M_u - M_{uv}}{C_w} = \frac{5477}{5.4} = 1014 \text{ kN}$$

The area of each boundary element is 21.40% of the total wall. The area of web is 57.2%.

Maximum compressive force on boundary element

$$= 0.214 \times 1.2(8189 + 4835) + 1014 = 4358 \text{ kN (compression)}$$

where load factor = 1.2, DL = 8189 kN, and LL = 4853 kN as per Table 21.4

Maximum tension force on boundary element

$$= 0.214 \times 0.8 \times (8189 + 4835) - 1014 = 1215 \text{ kN (compression)}$$

where load factor = 0.8 as per clause 9.4.3 of IS : 13920.

$\therefore$  There is no tension in the boundary element.

Axial load carrying capacity of boundary element with minimum 0.8% reinforcement

$$P_u = 0.4\sigma_{ck}A_c + 0.67\sigma_yA_{sc}$$

$$P_u = 0.4 \times 35 \times (600 \times 600 - 3216) + 0.67 \times 415 \times 3216$$

$$P_u = 5889 \text{ kN} > 4358 \text{ kN} \quad \text{OK}$$

$\therefore$  Minimum reinforcement 16-16 mm bars provided earlier in each boundary element is OK

### Confinement reinforcement in boundary element

Special confinement reinforcement should be provided throughout the height of boundary element as per clause 7.4.8. (clause 9.4.5)

$$\begin{aligned}
 A_{sh} &= 0.18 S_h \frac{\sigma_{ck}}{\sigma_{sp}} \left[ \frac{A_g}{A_c} - 1.0 \right] \\
 &= 0.18 \times S \times 125 \times \frac{35}{415} \left[ \frac{360000}{250000} - 1 \right] \\
 &= 0.8349 \times S \text{ mm}^2
 \end{aligned}$$

$h$  = longer dimension of the rectangular confining hoop measured to its outer face  
 $= (600 - 50 \times 2)/4 = 125 \text{ mm}$  along longitudinal direction

$S$  = spacing of hoops shall not exceed  
 ✦ 1/4 of minimum member dimension

✦ Need not be less than 75 mm nor more than 100 mm

$A_k$  = area of confined concrete core in the rectangular hoop measured to its outside dimensions.

$$= (600 - 100) \times (600 - 100) = 250000 \text{ mm}^2$$

$A_g$  = Gross area of boundary element

$$= 600 \times 600 = 360000 \text{ mm}^2$$

If  $S = 90 \text{ mm c/c}$ ,

$$A_{sh} = 75 \text{ mm}^2$$

∴ Provide 10 mm ties at 90 mm spacing in both directions.

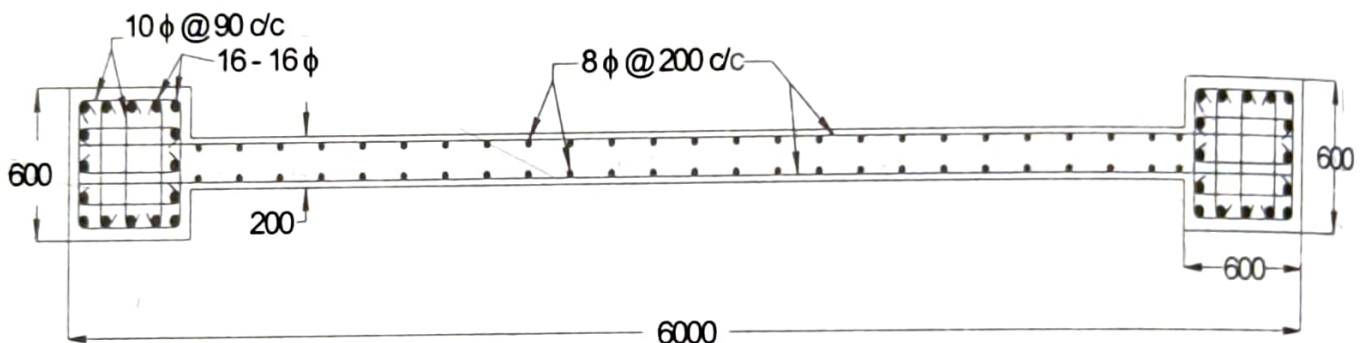


Fig. 21.19

### Variation of compressive stress ratio along the height

The continuity of boundary element is up to the height when the stress ratio in extreme fibre becomes less than 0.15. A check shows that the maximum axial stress ratio at the bottom is 0.547, at 12th storey is 0.15 and at top is 0.029. Thus, the boundary element should be continued up to 12th storey. The reinforcement details are shown in Fig. 21.19.

There is no need to provide special confinement steel in web of wall.